

UNIVERSITY OF RHODE ISLAND

Department of Mathematics and Applied Mathematical Sciences



Applied Mathematics and Scientific Computing Seminar

Location: Lippitt Hall 204 **Time:** Monday, December 9, 2024, 1:00pm (refreshments at 12:55 p.m.)

Gaussian Quadrature and the Tridiagonal Eigenvalue Problem

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Abstract: Quadrature methods are a way to approximate the definite integral of a function f(x) on the interval [a, b] by using a weighted sum of the form

$$\int_{a}^{b} f(x) \, dx \quad \approx \quad \sum_{j=0}^{n} w_{j} f(x_{j}) \, ,$$

where the x_j are n + 1 distinct points in [a, b] called the nodes, and the w_j are n + 1 positive real numbers called the weights.

Our presentation will begin by constructing this method. Through a nontrivial derivation, we will see that by choosing the nodes x_j and weights w_j strategically we can exactly integrate polynomials of degree much higher than n. This requires use of orthogonal polynomials and the tridiagonal matrix eigenvalue problem, leading to what is called *Gaussian* Quadrature.

Time permitting, we will introduce how the weights of Gaussian Quadrature can be used to accelerate convergence in some numerical linear algebra algorithms that approximate eigenvalues of a large matrix. More information on this will be presented in an upcoming seminar.

Graduate students, and undergraduate students with exposure to numerical analysis, are encouraged to attend.