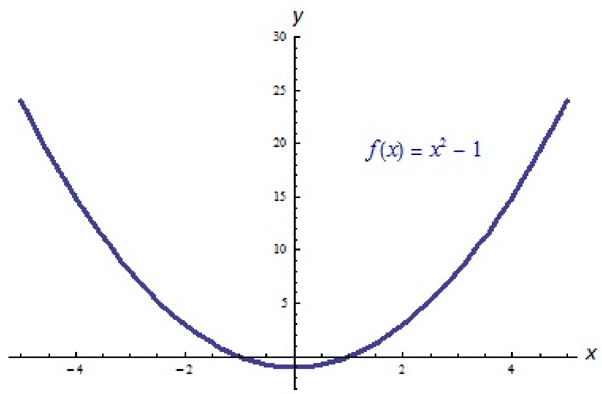
Lesson 1.7: *Introduction to Continuity*

Roughly speaking, a function is continuous on an interval if its graph has no breaks or jumps. A more intuitive way of saying this is that we can draw the graph of a continuous function in one go, without having to leave the paper.

Three Examples:

Example 1:

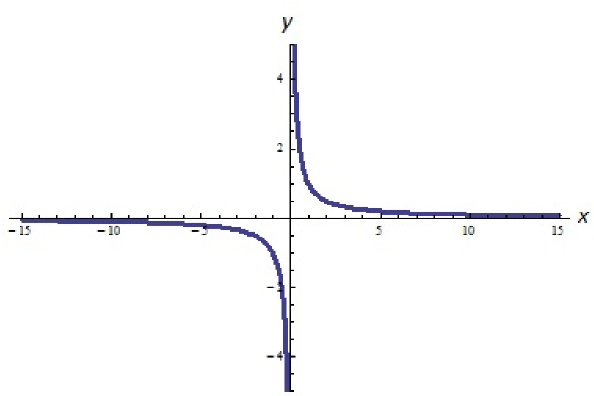
$$f(x) = x^2 - 1$$



Appears continuous on all of $\mathbb{R}$.

Example 2:

$$g(x) = \frac{1}{x}, x \neq 0$$

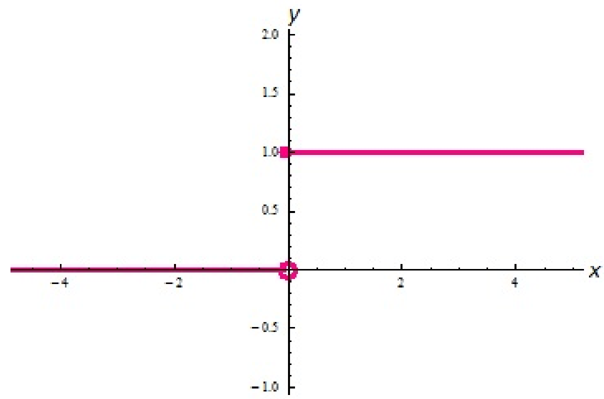


Not defined at 0, but looks continuous on any interval not containing 0.

Example 3:

$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

Heaviside Step Function



H has a jump at 0, but appears to be continuous on any interval not containing 0.

Which Functions are continuous?

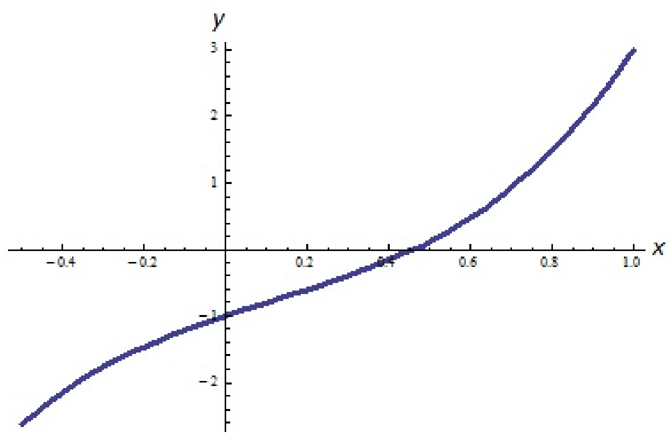
All polynomials, exponential functions, ln, sin, and cos are continuous on all of $\mathbb{R}$.

Any rational function function, $r(x) = \frac{P(x)}{Q(x)}$, is continuous on an interval for which the denominator, $Q(x)$, does not vanish.

Sums, differences and products of continuous functions are continuous as are quotients on intervals were the denominator doesn't vanish.

Compositions of continuous functions are continuous.

Consider $f(x) = 3x^3 - x^2 + 2x - 1$. $f(0) = -1$ and $f(1) = 3$. It seems reasonable (just by looking at the graph) to conclude that there is some value of x between 0 and 1 for which $f(x) = 0$.



This is in fact true, and the reason is that all continuous functions have the following property:

Theorem 1.1: Intermediate Value Theorem

Suppose f is continuous on a closed interval $[a, b]$. If k is any number between $f(a)$ and $f(b)$, then there exists $c \in [a, b]$ such that $f(c) = k$.