TWO EXAMPLES

Example 1:

A colony of bacteria doubles in size every 24 hours. If we start with one bacterium, find a formula for the population of bacteria at any given (subsequent) time.

Start by making a table:

t in days	number of bacteria
0	1
1	2
2	4
3	8
$\vdots$	$\vdots$

From this, we see that the function, $p(t) = 2^t$, (t in days) is suitable.

Example 2:

A radioactive sample has a half-life of two days. If we start off with N_0 atoms, find a formula for the number of (radioactive) atoms at any subsequent time.

Table:

t in days	number of radioactive atoms
0	N_0
2	$N_0/2 = \frac{N_0}{2}$
4	$N_0/4 = \frac{N_0}{2^2} = \frac{N_0}{2^{4/2}}$
6	$N_0/8 = \frac{N_0}{2^3} = \frac{N_0}{2^{6/2}}$

From this we see that the function,

$$\begin{aligned} N(t) &= N_0/2^{2/t} \\ &= N_0(\frac{1}{2})^{\frac{t}{2}} \\ &= N_0 2^{-\frac{t}{2}} \\ &= N_0(\frac{1}{\sqrt{2}})^t \end{aligned}$$

works.

Functions of the form $f(t) = ka^t$, $a > 0, k$ any real number, are called exponential functions.

If $k > 0$ and $a > 1$, we have exponential growth, as in Example 1, $(k = 1, a = 2)$, and the function is increasing.

If $k > 0$ and $a < 1$ we have exponential decay as in Example 2, $(k = N_0, a = \frac{1}{\sqrt{2}})$, and the function is decreasing.

Exponential Functions with base e

A very special number in connection with exponential functions is $e = 2.71828\ldots$. Just why e is special will be explain later. For now we note (without proof) that:

- Any exponential growth function, $p(t) = p_0 a^t, a > 1$, can be expressed as $p(t) = p_0 e^{kt}$ for a suitable positive constant k (called the growth constant)

- Any exponential decay function, $Q(t) = Q_0 a^t, a < 1$, can be expressed as $Q(t) = Q_0 e^{-kt}$, for another suitable positive constant k (called the decay constant)

Concavity

For a function, f, we say the graph of f is concave up if it bends upward as we

move left to right,

For a function, f, we say the graph of f is concave down if it bends downward as

we move left to right,

e.g: $e^x, x^2, \frac{1}{2^x}$, are all concave up. $f(x) = 1 - e^{-x}$ is concave down.