A function allows us to express just how one quantity depends on another.

Example: Hubble's Law

Edwin Hubble (1889-1953) showed that the Milky Way was not the only galaxy in our universe. Observing these other galaxies, he noticed (by means of spectral redshift) that they all seemed to me moving away from us. Moreover, the more distant the galaxy, the faster it was receding. Eventually, he came up with the following:

$$v = H_0 D$$

where v is the recession velocity in km/sec, D is the distance in megaparsec, (1 megaparsec=3.26 million light years) and H_0 is called **Hubble's constant**.

Note: **ALWAYS REMEMBER TO USE THE CORRECT UNITS!!!**

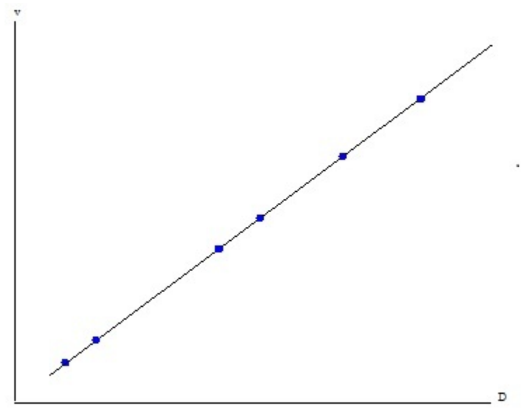
Determining its exact value is one of the great challenges of modern cosmology. The currently accepted value of H_0 is $70\frac{km/sec}{megaparsec}, (+2.4, -3.2)$.

n.b: The above relationship is only approximate for any given specific galaxy (peculiar motion).

In practice, what Hubble did was first enter his observational data in a table something like this:

D (mpsec)	1	1.5	3	3.5	4	5
v (km/sec)	70	105	210	245	280	350

From this he could compile a graph:



Using this data, Hubble was able to pass a line through the points on the graph and *deduce* the formula:

$$v = H_0 D$$

So far, we have seen three ways of representing a function: tables, graphs and formulae. A fourth way is in words. e.g: The speed with which a galaxy is receding from us is directly proportional to its distance, the constant of proportionality being Hubble's constant of $70\frac{km/sec}{mpsec}$!

Consequences of Hubble's Law:

- If we know how far a galaxy is away from us, we can estimate how fast it is receding (and vice versa).
- The universe is expanding!
- Hubble's law places constraints on the overall shape (curvature) of the universe.

This is a good illustration of the power of functions when it comes to answering deep questions about our world.

Def'n: A function is a rule (often a formula) which for each number in a certain set of input numbers (called the *domain*), gives us a number in a set of output numbers (called the *range*).

Numbers:

Natural Numbers: $\mathbb{N} = \{1, 2, 3, 4, \dots\}$

Integers: $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Rational Numbers: $\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z}, p \neq 0\}$

Irrational Numbers : <u>e.g</u> $\sqrt{2}, \pi, e, ect.$

Real Numbers: $\mathbb{R} = \mathbb{Q} \cup Irrational Numbers$

Intervals:

$(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$, **open interval**

$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$, **closed interval**

$(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$

$[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$

$(a, \infty) = \{x \in \mathbb{R} \mid a < x\}$

$[a, \infty) = \{x \in \mathbb{R} \mid a \leq x\}$

$(-\infty, b) = \{x \in \mathbb{R} \mid x < b\}$

$(-\infty, b] = \{x \in \mathbb{R} \mid x \leq b\}$

A WORD ON DOMAIN AND RANGE:

Usually, if we don't explicitly specify the domain of a function we take it to be the largest possible set of real numbers for which the function makes sense (maximal domain).

e.g

$f(x) = x^2$ is defined on all of $\mathbb{R}$

$g(x) = \frac{1}{x}$ is defined on $\mathbb{R}\backslash\{0\}$

$h(x) = \sqrt{x}$ is defined on $\{x \mid x \geq 0\} = [0, \infty)$

Sometimes we restrict the domain

e.g:

$p(x) = x^2$ defined on $[0, \infty)$, (range $=[0, \infty)$ also).

`Question:` What are the domain and range for the function in Hubble's Law?

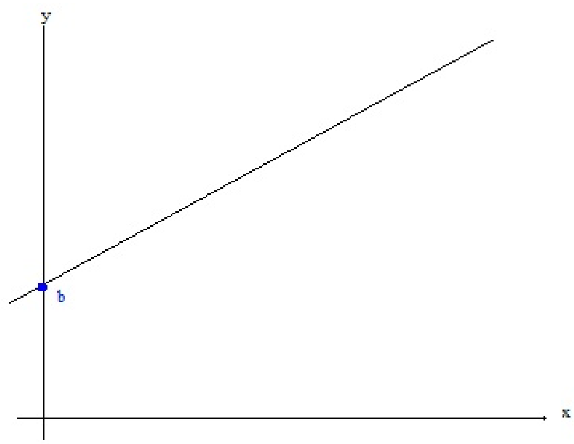
Linear Functions:

Hubble's law is an example of a linear function. In general, a linear function is of the form

$$y = mx + b$$

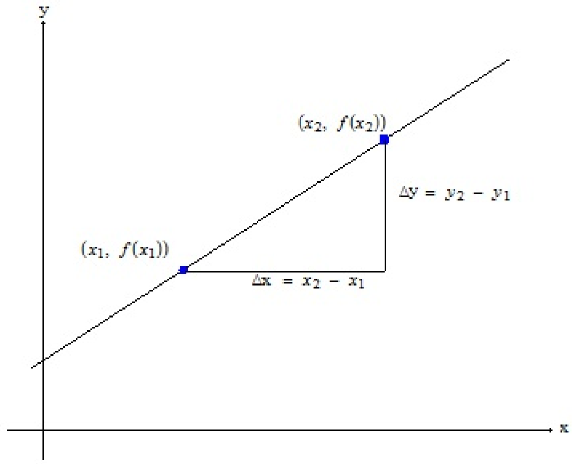
where

- m is the slope, or the rate of change of y with respect to x
- b is the vertical intercept, or the value of y when x is zero.



Given two different values of x, x_1 and x_2 and two corresponding points $(x_1, f(x_1)), (x_2, f(x_2))$ on the graph of f, we can calculate the slope m by

$$\begin{aligned} m &= \frac{\text{change in y}}{\text{change in x}} \\ &= \frac{\triangle\, y}{\triangle\, x} \\ &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \end{aligned}$$



The slope m measures how fast the function increases or decreases. In general, for a function, $f(x)$, we say that f is increasing if the values of $f(x)$ increase as x increases, f is decreasing if the values of $f(x)$ decrease as x decreases.

If f is a linear function, $f(x) = mx + b$, then f is increasing if and only if $m > 0$, and f is decreasing if and only if $m < 0$.

`Question:` What does the value (and sign) of Hubble's Constant tell us about the universe?