Math 141 Final Worksheet #2

1. Differentiate each of the following

i) $f(x) = \arctan(x^7)$

ii) $g(x) = \ln(\arcsin(2x))$

iii) $h(x) = e^{x^2} \tan^2 x$

iv) $k(x) = \dfrac{1 + \sec x}{1 - \sec x}$

v) $l(x) = 2^{\arcsin(x^3)}$

2. Use implicit differentiation to find $\frac{dy}{dx}$ for the relation

$$\sin(x+y) = (x-y)e^{y^2}.$$

3. The number of eggs, E, a farmer sells each week depends on the price p, in dollars, of the chicken feed.

a) Give the units of $E'(p)$.

b) Explain the statements $E(100) = 342$ and $E'(100) = -2$.

4. a) Find the tangent line approximation for $f(x) = \arctan x$ near $x = \frac{\pi}{4}$.

b) Use a) to approximate $\arctan \frac{\pi}{3}$.

c) Is the answer from b) larger or smaller than the actual value of $\arctan \frac{\pi}{3}$? Explain (without resorting to your calculators!).

5. Let $f(x) = x^4 - 4x^3 + 4x^2 + 1$

a) Find all the critical points of f and use the first derivative test to classify their type.

b) Find all points of inflection of f.

c) Sketch the graph of f.

d) Find the max and min of f on the interval $[.5, 3]$.

6. A rectangular swimming pool is to be built with an area of $1800 ft^2$. The owner wants 5 ft wide decks along either side and 10 ft wide docks at the two ends. Find the dimensions of the smallest piece of property on which the pool can be built satisfying these conditions.

7. A piece of steel is being squeezed in a hydraulic press. If the steel preserves the shape of a (flattening) cylinder and has a constant volume of 2000π cm^3, how fast is the radius growing when the height of the cylinder is 5cm given that the press is descending at 1cm/min at this point?

8. Calculate each of the following limits using l'Hôpital's rule.

i) $\lim_{x \to 0} \frac{\sin^2(2x)}{3x^2}$

ii) $\lim_{x \to -\infty} \frac{x^2}{e^{-x}}$

iii) $\lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1}$

iv) $\lim_{x \to 0} \frac{1}{x} - \frac{1}{\sin x}$