Chapter 6 - Constructing Antiderivatives

§ 6.1 Antiderivatives Graphically and Numerically

Recall that for a fn f, another fn F is an antiderivative of f if

$$F'(x) = f(x).$$

Going from f to F is the 'opposite' of differentiation.

Any function f has many antiderivatives.

e.g. x^2 is an antid of $2x$, but so is x^2+1, x^2-7 and in general x^2+C for any constant C. Say the fn $2x$ has a family of antiderivatives (antids.).

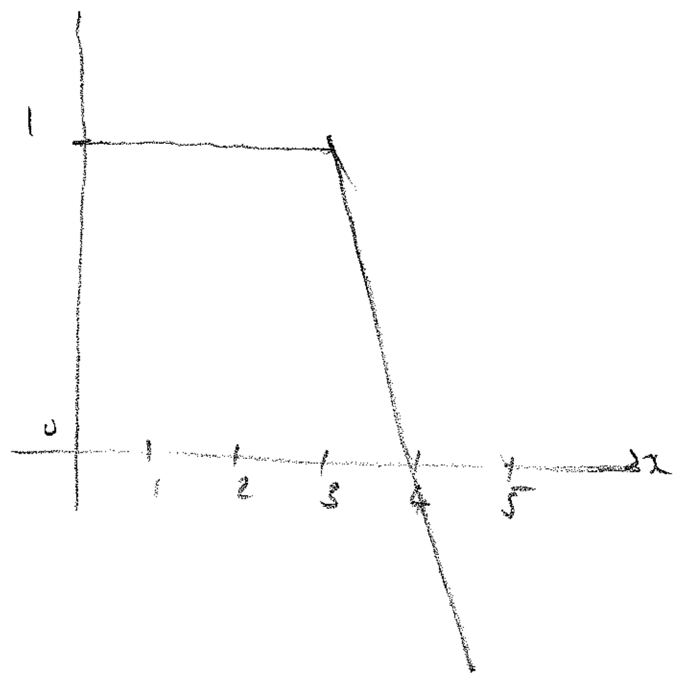
Visualising Antiderivatives Using Slopes

Suppose we have the graph of f' and we want to sketch an approximate graph of f.

Looking for a graph of f whose slope at any pt. is equal to the value of f' there.

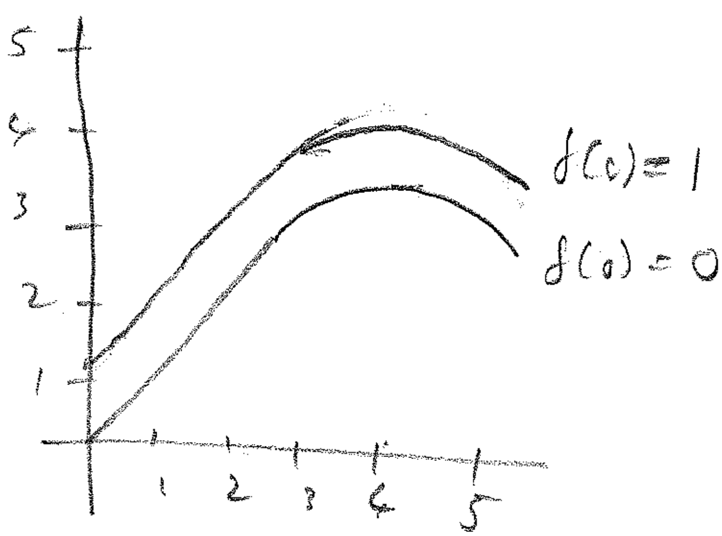
- Where f' is above the x-axis, f is incr.
- Where f' is below the x-axis, f is decr.
- Where f' is incr, f is concave up
- Where f' is decr., f is concave down.

Ex 1. For a fn f' whose graph is as below, sketch a graph of f in the cases when $f(0)=0$ & $f(0)=1$



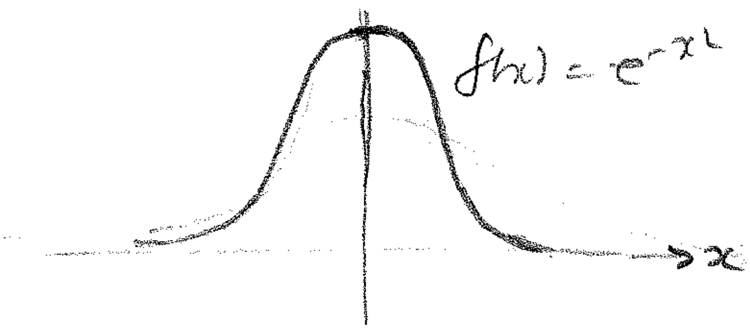
For $0 \le x \le 2$, f has a const. slope of 1, so the graph of f is a straight line.

For $3 \le x \le 4$, f is incr but more slowly. f has a max at 4 and for $4 \le x \le 5$, f is decr.

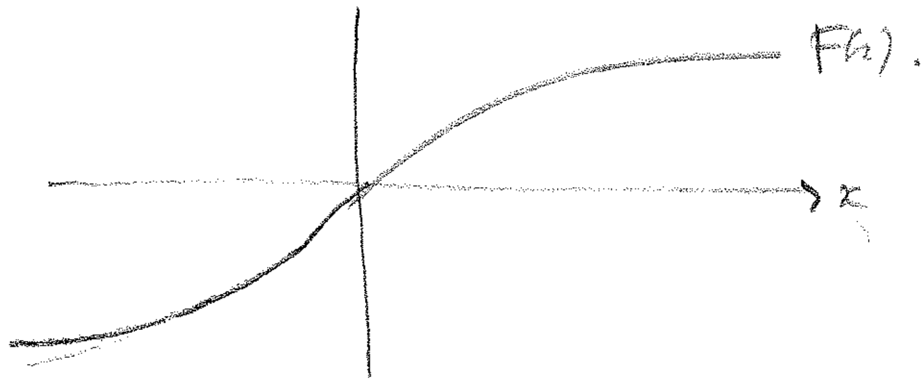


Ex2. Sketch a graph of the antid. F of $f(x) = e^{-x^2}$ satisfying $F(0) = 0$.

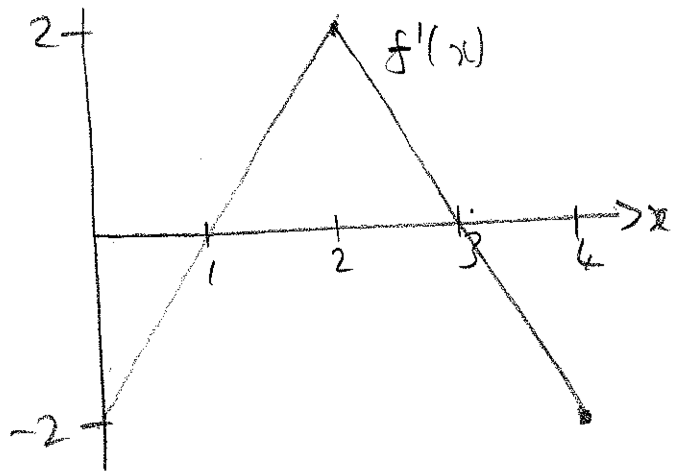
Graph of e^{-x^2} looks like



Slope of antid F is given by $f(x) = e^{-x^2}$.
Since this fn is always >0, F is always incr. Hence since $F(0)=0$, F is <0 for $x<0$ and >0 for $x>0$.
Since f is incr for $x<0$, F is concave up for $x<0$
Since f is dec for $x>0$, f is concave down for $x>0$.
Finally, since $f(x) \to 0$ as $x \to \pm\infty$
the graph of $F(x)$ levels off at both ends.



Ex 3. For the fn f' given below, sketch a graph of an antid. f with $f(0)=0$.

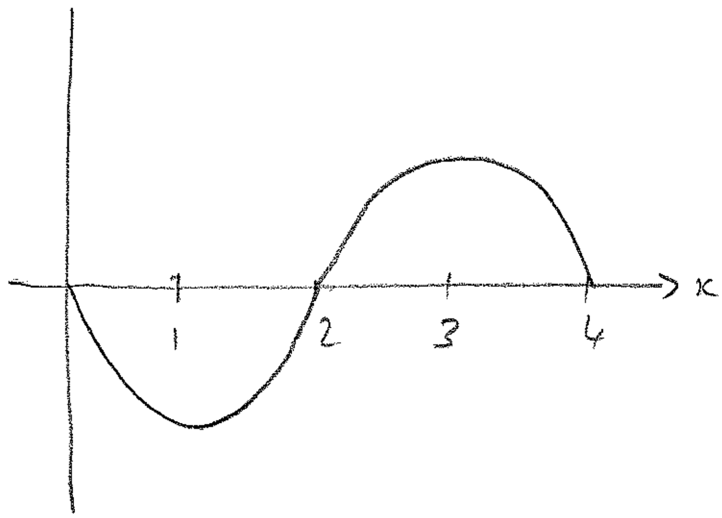


$f'<0$ for $0<x<1$,
$3<x<4$,
so f should be decr. here.

$f'>0$ for $1<x<3$,
so f should be incr here.

$f'(x)$ is incr for $0<x<2$, so f should be concave up here.

$f'(x)$ is decr for $2<x<4$, so f should be concave down here.



Computing Values of an Antiderivative Using the Fundamental Theorem.

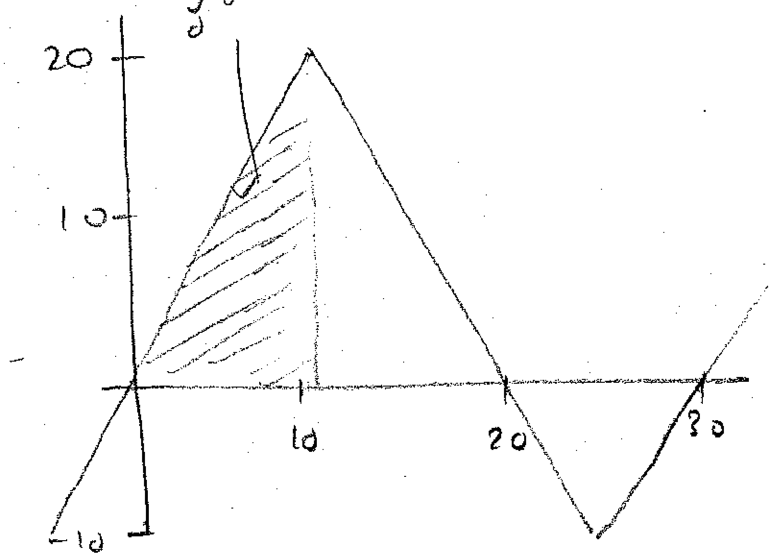
Can use

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

to find $f(b)$ if we know $f'(x)$ and $f(a)$.

<u>Ex 4</u>. The graph shows the derivative $f'(x)$ of a f^n f for which $f(0) = 100$. Sketch the graph of f, showing all crit pts. and pts. of infl.

shaded $- \int_0^{10} f(t)\,dt$



Crit pts occur when $f' = 0$ $(0, 20, 30)$.

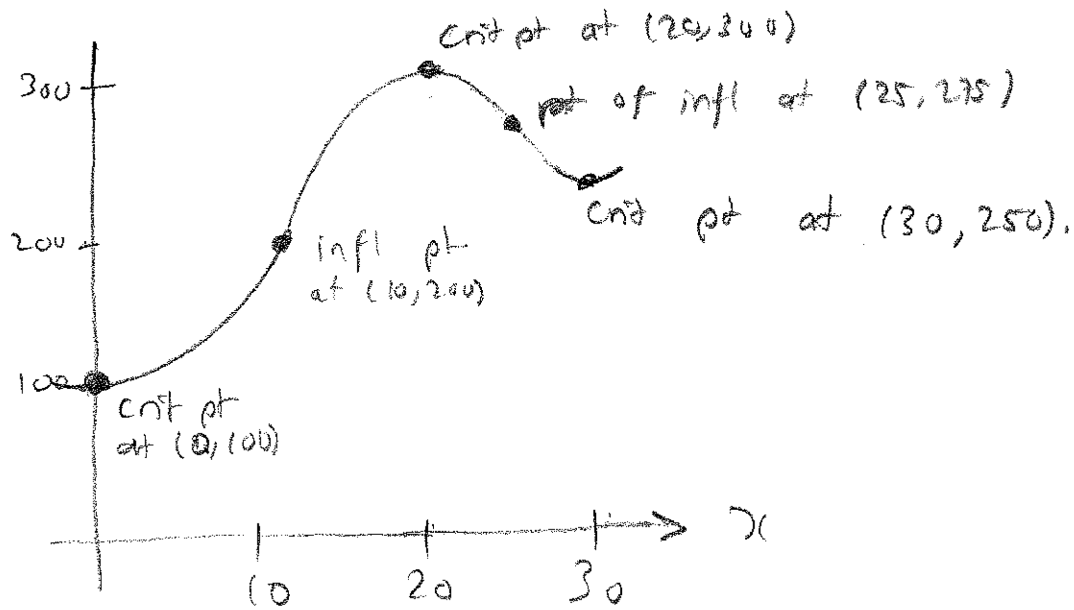
Infl. pts. are when the concavity changes, ie when f' changes from incr. to decr. or vice versa.

Can work out values of f from definite integrals using FTOC & the definite integrals can be calculated using area.

e.g. $f(10) - f(0) = \int_0^{10} f'(x)\,dx = \frac{1}{2}\, 10 \times 20 = 100$

$f(10) - 100 = 100$

So $f(10) = 200$.



$$f(20) = f(10) + \int_{10}^{20} f'(x)\,dx$$

$$= 200 + \frac{1}{2} \times 20 \times 10 = 300$$

$$f(25) = f(20) + \int_{20}^{25} f'(x)\,dx$$

$$= 300 - \frac{1}{2} \times 5 \times 10 = 275$$

$$f(30) = f(25) + \int_{25}^{30} f'(x)\,dx$$

$$= 275 - \frac{1}{2} \times 5 \times 10 = 250.$$

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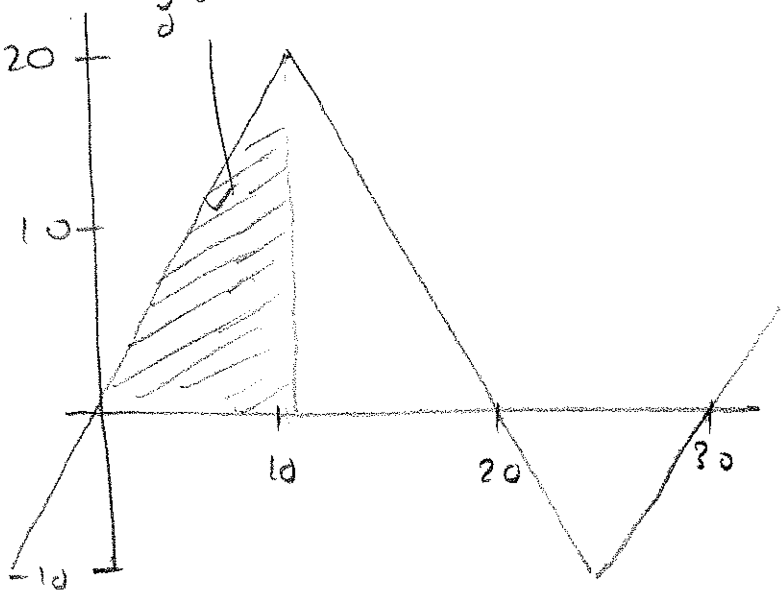

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