

## § 5-1 Distance / Area

For an object at constant velocity,  
we have

$$\text{displacement} = \text{velocity} \times \text{time}.$$

Now suppose we have non-const vel.

e.g. an accel. car where we measure vel.  
every two seconds

|           |    |    |    |    |    |    |
|-----------|----|----|----|----|----|----|
| $t(s)$    | 0  | 2  | 4  | 6  | 8  | 10 |
| $v(ft/s)$ | 20 | 30 | 38 | 44 | 48 | 50 |

Get an approx dist. of

$$2 \times 30 + \dots + 2 \times 48 = 360 \text{ ft}$$

This is an under-estimate. Overestimate is 420 ft.  
So  $360 \text{ ft} < \text{true dist} < 420 \text{ ft}.$

However, this can only approx the  
true displ. as the vel. is changing.

If we measure vel. every second instead, we should get a better approx to the true dist travelled.

|           |    |    |    |    |    |    |    |    |    |    |    |
|-----------|----|----|----|----|----|----|----|----|----|----|----|
| $t(s)$    | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| $v(ft/s)$ | 20 | 26 | 30 | 34 | 38 | 41 | 44 | 46 | 48 | 49 | 50 |

New lower est. is

$$20 \times 1 + 26 \times 1 +$$

$$+ 49 \times 1 = 376 \text{ ft}$$

(note  $> 360 \text{ ft}$ )

New upper est is

$$26 \times 1 +$$

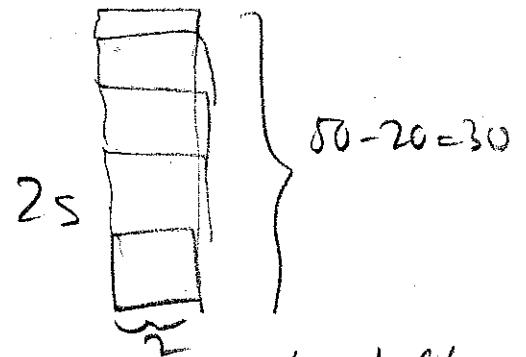
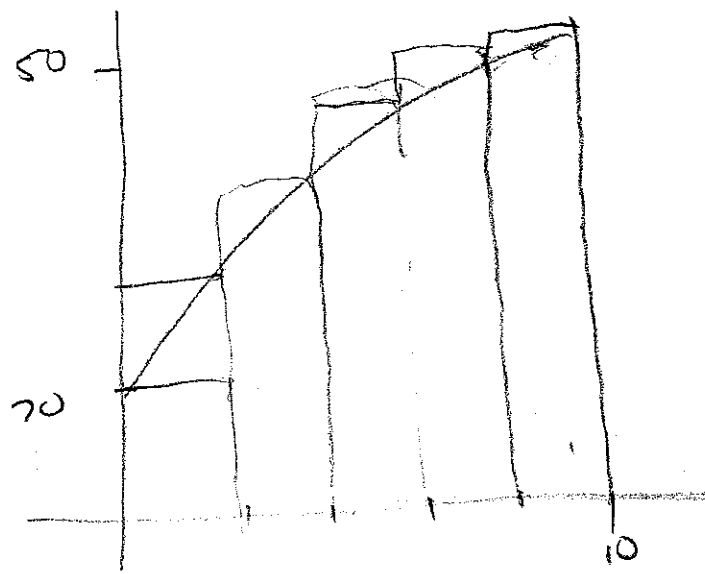
$$+ 50 \times 1 = 406 \text{ ft}$$

(note  $\leq 420 \text{ ft}$ )

So we now know.

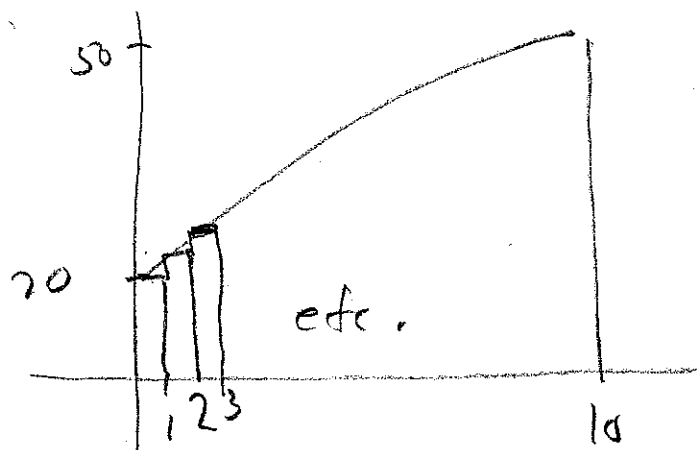
$$376 \text{ ft} < \text{Actual dist} < 406 \text{ ft.}$$

Of course, if we measured ever  $\frac{1}{2} \text{ s}$ ,  $\frac{1}{4} \text{ s}$  etc., we would expect to get even better approxs.



Note total diff  
between ests

$$\text{is } 30 \times 2 = 60$$



$$30 \times 1 = 30.$$

What we are in effect doing is trying  
to find the area under the graph of  
vel. against time.

Ex 1. What is the diff. between the upper & lower est if the vel. were given every

a) .1 s - Ans.  $(50 - 20) \times .1 = 3 \text{ ft}$

b) .01 s - Ans.  $(50 - 20) \times .01 = .3 \text{ ft.}$

Ex 2 How frequently should we measure vel. if we want to get the total dist travelled to within .1 ft?

$$(30)h < .1$$

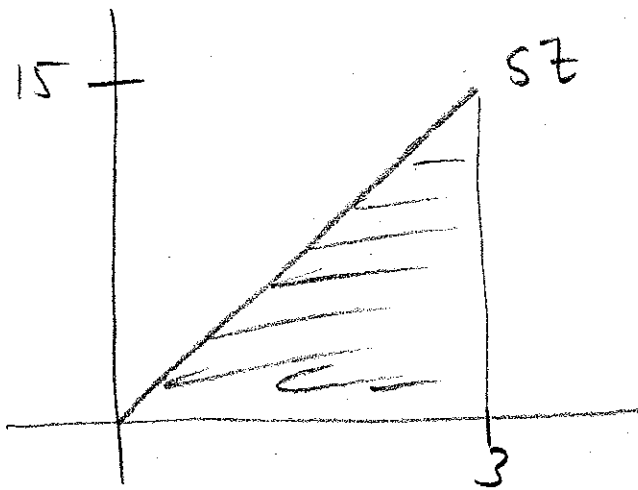
$$h < \frac{.1}{30} = 0.0033$$

(300 times a s.)

So if measurements are made .0033 s. apart, the estimate is accurate to within 1 ft.

Ex 2 With time  $t$  in s, the vel. of a bicycle in ft/s. is given by  $v(t) = 5t$ . How far does the bicycle travel in 3s?

Soln. vel. is linear



Dist. is area of a triangle of height 15 & base 3.

Hence

$$\text{Dist} = \frac{1}{2} 3 \times 15 = 22.5 \text{ ft.}$$

Ex. A particle moves along the y-axis with vel. 30 cm/s for 5s and vel. -10 cm/s for the next 5s.

the vel represents upward motion and -ve vel. ——— downward motion.

What is represented by the sum

$$30 \cdot 5 + (-10) \cdot 5?$$

