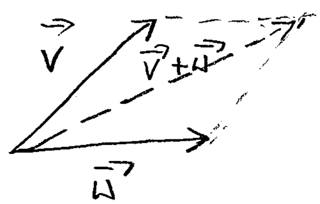
13.1 – 13.2 Examples

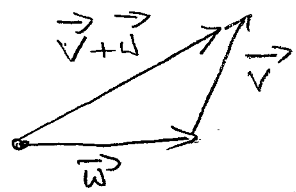
Examples are selected from Handout 8.

We are working with vectors interpreted purely geometrically as arrows and with vectors represented in terms of their $\vec{i}, \vec{j}, \vec{k}$ components.

We define operations on vectors : $\vec{v}+\vec{w}$, $\|\vec{v}\|$, $\alpha\vec{v}$, $\vec{v}-\vec{w}$ in purely geometric terms as follows. Let two vectors $\vec{v}$, $\vec{w}$ and a scalar α be given. Then the sum $\vec{v}+\vec{w}$ is

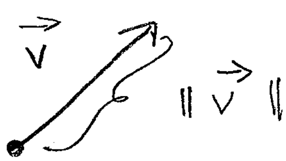


or

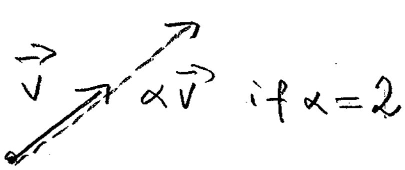


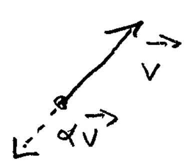
(Remember the tail of a vector doesn't matter; we can move it freely.)

The magnitude $\|\vec{v}\|$ is the length of the arrow representing $\vec{v}$. In other words, the distance between the tail and the endpoint of $\vec{v}$:



$\alpha\vec{v}$ is a vector parallel to $\vec{v}$ with its length rescaled by the factor α. If $\alpha > 0$, $\alpha\vec{v}$ points in the same direction as $\vec{v}$, if $\alpha < 0$, $\alpha\vec{v}$ points in the opposite direction.

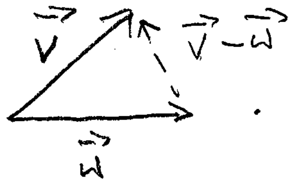




if $\alpha = -\frac{1}{2}$.

$$\vec{v} - \vec{w} = \vec{v} + (-\vec{w}), \quad \text{where } -\vec{w} = -1 \cdot \vec{w}.$$

Geometrically:



Ex 1: The vectors $\vec{w}$ and $\vec{u}$ are in Figure 12.16. Match the vectors $\vec{p}, \vec{q}, \vec{r}, \vec{s}, \vec{t}$ with five of the following vectors: $\vec{u}+\vec{w}$, $\vec{u}-\vec{w}$, $\vec{w}-\vec{u}$, $2\vec{w}-\vec{u}$, $\vec{u}-2\vec{w}$, $2\vec{w}$, $-2\vec{w}$, $2\vec{u}$, $-2\vec{u}$, $-\vec{w}$, $-\vec{u}$.

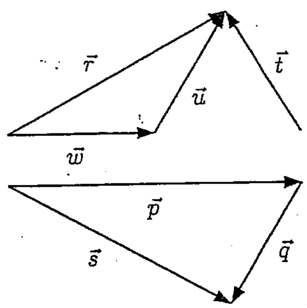


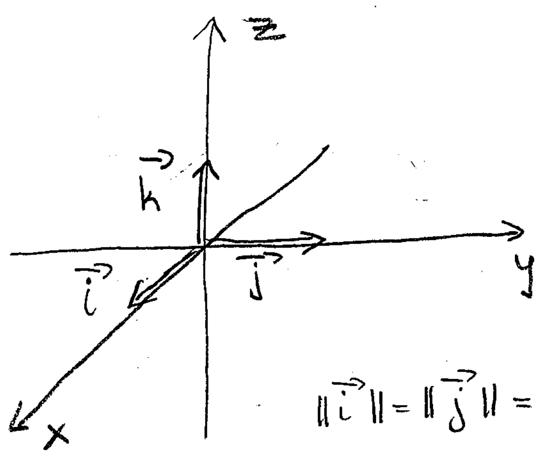
Figure 12.16

$\vec{r} = \vec{w} + \vec{u}$, $\vec{t} = \vec{u} - \vec{w}$

$\vec{p} = 2\vec{w}$, $\vec{q} = -\vec{u}$

$\vec{s} = \vec{p} + \vec{q} = 2\vec{w} - \vec{u}$

We choose basic vectors $\vec{i}, \vec{j}, \vec{k}$ in the xyz-space as follows:



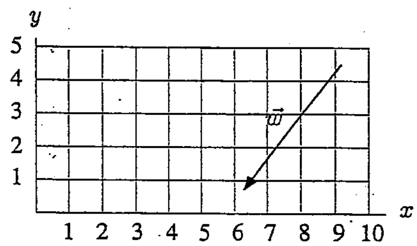
$$\|\vec{i}\| = \|\vec{j}\| = \|\vec{k}\| = 1$$

Every vector $\vec{v}$ in the xyz-space can be uniquely represented as:

$$\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}.$$

The numbers v_1, v_2, v_3 are called components of $\vec{v}$

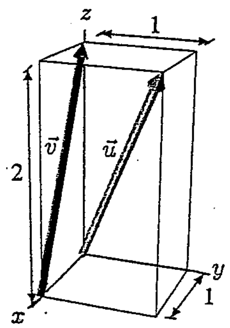
Ex 2 : Resolve the vector $\vec{w}$ below into its $\vec{i}, \vec{j}$ components : $\vec{w} = w_1 \vec{i} + w_2 \vec{j}$:



$\vec{w}$ displaces its tail about 2.5 units in the negative $\vec{i}$ direction and about 3.5 units in the negative $\vec{j}$ direction.

Hence : $\vec{w} = -2.5\vec{i} - 3.5\vec{j}$

Ex 3 : Resolve $\vec{u}$ and $\vec{v}$ into their $\vec{i}, \vec{j}, \vec{k}$ components :



Looking at how many units $\vec{u}$ displaces in the $\vec{i}, \vec{j}, \vec{k}$ directions, we see that :

$$\vec{u} = \vec{i} + \vec{j} + 2\vec{k}$$

Similarly,

$$\vec{v} = -\vec{i} + 2\vec{k} .$$

All operations can easily be expressed in terms of components. Let $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$, $\vec{w} = w_1\vec{i} + w_2\vec{j} + w_3\vec{k}$ be given. Then:

$$\vec{v} + \vec{w} = (v_1 + w_1)\vec{i} + (v_2 + w_2)\vec{j} + (v_3 + w_3)\vec{k},$$

$$\vec{v} - \vec{w} = (v_1 - w_1)\vec{i} + (v_2 - w_2)\vec{j} + (v_3 - w_3)\vec{k},$$

$$\alpha\vec{v} = (\alpha v_1)\vec{i} + (\alpha v_2)\vec{j} + (\alpha v_3)\vec{k} \quad \text{for any scalar } \alpha,$$

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2} .$$

We define the zero vector $\vec{0}$ as:

$$\vec{0} = 0\vec{i} + 0\vec{j} + 0\vec{k}.$$

A vector $\vec{u}$ is called a unit vector if $\|\vec{u}\| = 1$.

Observe the following:

$$\|\alpha\vec{v}\| = |\alpha|\,\|\vec{v}\|$$

for every vector $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$ and scalar α. Of course:

$$\|\alpha\vec{v}\| = \|\alpha v_1\vec{i} + \alpha v_2\vec{j} + \alpha v_3\vec{k}\| = \sqrt{\alpha^2 v_1^2 + \alpha^2 v_2^2 + \alpha^2 v_3^2} =$$

$$= \sqrt{\alpha^2}\sqrt{v_1^2 + v_2^2 + v_3^2} = |\alpha|\,\|\vec{v}\|.$$

Ex 4: Let $\vec{v} = \vec{i} - \vec{j} + 2\vec{k}$. Find the unit vector $\vec{u}$ in the direction of $\vec{v}$.

Note the $\frac{1}{\|\vec{v}\|}\vec{v}$ points in the direction of $\vec{v}$. Also,

$$\left\| \frac{1}{\|\vec{v}\|}\vec{v} \right\| = \frac{1}{\|\vec{v}\|}\|\vec{v}\| = 1.$$ So we can take

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}.$$

$\|\vec{v}\| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$. Hence $\vec{u} = \frac{1}{\sqrt{6}}\vec{i} - \frac{1}{\sqrt{6}}\vec{j} + \frac{2}{\sqrt{6}}\vec{k}$.