§ 3.3 Continuous Random Variables

Many of the quantities one wants to measure vary continuously e.g. the time of day, the height of a randomly chosen URI student. Hence we need continuous random variables which can take on a whole range (interval) of values (a formal defn will be given later).

<u>Ex 3.8</u> Accidents on a 200km long stretch of freeway.

. Assume accidents are equally likely at all places on the freeway.

Suppose there is an accident.

The prob. the accident occurred in the first 100km of freeway is $\frac{1}{2}$.

The prob the accident occurred between 100km and 150km is $\frac{1}{4}$.

Our sample space is, of course, the interval $[0, 200]$ and we will say the prob. the accident occurred in a stretch of freeway d km in length is $\frac{d}{200}$.

Q. Does this lead to a consistent way to assign probability which satisfies the 3 postulates of probability?

A. Yes, but technical (measure theory).

We can check some things, however.

For P2, $P(S) = P([0, 200]) = \frac{200}{200} = 1$.

For P3, if $I_1, I_2, I_3, \_\_$ is a countable collection of pairwise disjoint intervals of length $d_1, d_2, d_3, \_\_$, we can define

$$P(\bigcup_i I_i) = \sum_i \frac{d_i}{200}$$

and this will work.

Finally P1 is obvious for intervals and by our extension to countable collections of pairwise disjoint intervals.

Note: The prob. that the accident occurred on a very short stretch of freeway is very low.

e.g. $P([100, 100.1]) = 0.0005$ $(= .05\%)$

and if we take smaller and smaller intervals, we see that $P(\{100\}) = 0$ and indeed $P(\{x\}) = 0$ for any $0 \leq x \leq 200$.

THIS IS NOT A CONTRADICTION!

There are 'a lot' of points in $[0, 200]$ (ie uncountably many).