## Mth 435 Homework 1

Dr. Mark Comerford
Due Wednesday September 20, 2013
All problems are worth 20 points, including bonus problems which are extra credit.

1. Symbolize the following sentence: 'If $2+2=4$, then the sun rises in the East'.
Is this a true statement? Describe what might be considered unsatisfactory about such a statement.
2. Symbolize the following: 'Every even integer greater than 2 can be expressed as the sum of two primes'.

Write down the logical negation of this statement, both symbolically and as a sentence in ordinary English.
n.b. This is the famous Goldbach Conjecture!
3. Let $A, B$ be sets and $f: A \mapsto B$ be a function. Show that for subsets $C, D$ of $A$,
(a) $f(C \cup D)=f(C) \cup f(D)$,
(b) $f(C \cap D) \subseteq f(C) \cap f(D)$.

For (b), provide a counterexample for which the sets $f(C \cap D)$ and $f(C) \cap f(D)$ are different.
4. Again let $A, B$ be sets and $f: A \mapsto B$ be a function. Recall that for a subset $E$ of $B$, the preimage or inverse image of $E$, written $f^{-1}(E)$ is given by

$$
f^{-1}(E):=\{a \in A: f(a) \in E\} .
$$

If $E$ and $F$ are subsets of $B$, show that
(a) $f^{-1}(E \cup F)=f^{-1}(E) \cup f^{-1}(F)$
(b) $f^{-1}(E \cap F)=f^{-1}(E) \cap f^{-1}(F)$
5. Prove by mathematical induction that for every natural number $n$

$$
1^{3}+2^{3}+3^{3}+\cdots \cdots+n^{3}=\left(\frac{1}{2} n(n+1)\right)^{2}
$$

6. Prove by mathematical induction that $n^{3}+5 n$ is divisible by 6 for any natural number $n$.
