# Mth 435 Homework 3 - Metric Spaces

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Due Friday November 16, 2012

**All problems are worth 20 points, including bonus problems which are extra credit.**

1. Let $(X, d)$ be a metric space and $E \subset X$. We define the *closure* of $E$

$$\overline{E} := \{x \in X : x \text{ is adherent to } E\}.$$

   Show the following:

   (a) $E \subset \overline{E}$,

   (b) $\overline{E}$ is closed,

   (c) $E$ is closed if and only if $E = \overline{E}$.

2. Let $f : \mathbb{R}^2 \mapsto \mathbb{R}$ be continuous. Show that any set of the form

$$\{(x, y) \in \mathbb{R}^2 : f(x, y) < c\}$$

   is open while any set of the form

$$\{(x, y) \in \mathbb{R}^2 : f(x, y) \leq c\}$$

   is closed. Use this to investigate which of the following sets are open, closed, or neither:

   (a) $\{(x, y) : x > 1/y\}$.

   (b) $\{(x, y) : y = \sin x\}$,

   (c) $\{(x, y) : |x| \leq 1 \bigwedge |y| \leq 1\}$.