

Mth 435 Homework 1

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Due Wednesday September 19, 2012

All problems are worth 20 points, including bonus problems which are extra credit.

1. Symbolize the following sentence: ‘If $2 + 2 = 4$, then the sun rises in the East’.

Is this a true statement? Describe what might be considered unsatisfactory about such a statement.

2. Symbolize the following: ‘Every even integer greater than 2 can be expressed as the sum of two primes’.

Write down the logical negation of this statement, both symbolically and as a sentence in ordinary English.

n.b. This is the famous Goldbach Conjecture!

3. Let A, B be sets and $f : A \mapsto B$ be a function. Show that for subsets C, D of A ,

(a) $f(C \cup D) = f(C) \cup f(D)$,

(b) $f(C \cap D) \subseteq f(C) \cap f(D)$.

For (b), provide a counterexample for which the sets $f(C \cap D)$ and $f(C) \cap f(D)$ are different.

4. Again let A, B be sets and $f : A \mapsto B$ be a function. Recall that for a subset E of B , the *preimage* or *inverse image* of E , written $f^{-1}(E)$ is given by

$$f^{-1}(E) := \{a \in A : f(a) \in E\}.$$

If E and F are subsets of B , show that

(a) $f^{-1}(E \cup F) = f^{-1}(E) \cup f^{-1}(F)$

(b) $f^{-1}(E \cap F) = f^{-1}(E) \cap f^{-1}(F)$

5. Prove by mathematical induction that for every natural number n

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{1}{2}n(n+1)\right)^2.$$

6. Prove by mathematical induction that $n^3 + 5n$ is divisible by 6 for any natural number n .