Math 215 Homework 1

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1.1.2 Solve using elementary row operations

$$\begin{aligned} 2x_1 + 6x_2 &= -4 \\ 5x_1 + 7x_2 &= 10 \end{aligned}$$

1.1.12 Solve using row operations on the augmented matrix

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned}$$

1.1.16 Determine if the following system is consistent *without solving it.*

$$\begin{aligned} x_1 - 6x_2 &= 5 \\ x_2 - 4x_3 + x_4 &= 0 \\ -x_1 + 6x_2 + x_3 + 5x_4 &= 3 \\ -x_2 + 5x_3 + 4x_4 &= 0 \end{aligned}$$

Please turn over!

1.2.4 Row reduce the following matrix to reduced echelon form.

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}$$

1.2.12,13 Find the general solutions of the systems whose augmented matrices are as below.

$$\left[\begin{array}{cccc|c} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right] \qquad \left[\begin{array}{ccccc|c} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

1.2.31 A system of linear equations with more equations than unknowns is sometimes called an *overdetermined system*. Can such a system be consistent? Illustrate your answer with a specific system of three equations in two unknowns.

1.3.1,2 Compute $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - 2\mathbf{v}$ for the following examples:

$$\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

Please turn over!

1.3.10 Write a vector equation which is equivalent to the following system of equations.

$$\begin{aligned} 3x_1 - 2x_2 + 4x_3 &= 3 \\ -2x_1 - 7x_2 + 5x_3 &= 1 \\ 5x_1 + 4x_2 - 3x_3 &= 2 \end{aligned}$$

1.3.165 (Similar to 1.3.18 in 3rd Ed.) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}.$$

For what values of h is $\mathbf{y}$ in the plane spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$?

1.3.25 Let

$$A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}.$$

Denote the columns of A by $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, and let $W = \text{Span}[\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3]$.

a. Is $\mathbf{b}$ in $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$? How many vectors are in $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$?

b. Is $\mathbf{b}$ in W?

c. Show that $\mathbf{a}_1$ is in W. *Hint:* Row operations are unnecessary here!

Please turn over!

1.4.12 Let

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}.$$

Write the augmented matrix for the linear system which corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$. Then solve the system and write the solution as a vector.

1.4.13 Let

$$\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$$

Is $\mathbf{u}$ in the plane in $\mathbb{R}^3$ spanned by the columns of A? Justify your answer!

1.4.17 Let

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$$

Does A have a pivot position in every row? *Hint:* Recall that a pivot position in a matrix A is a location in A that corresponds to a 1 (in the same position) in the reduced echelon form of A.

Please turn over!

1.4.18 Let

$$B = \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{bmatrix}$$

Do the columns of B span $\mathbb{R}^4$? *Hint:* For this you will need a theorem from the notes which you should cite. The previous question may also be of help!

1.5.3 Determine if the following system has a solution. You should try to use as few row operations as possible.

$$\begin{aligned} -3x_1 + 4x_2 - 8x_3 &= 0 \\ -2x_1 + 5x_2 + 4x_3 &= 0 \end{aligned}$$

1.5.5 Write the solution of the following homogeneous system in parametric form.

$$\begin{aligned} 2x_1 + 2x_2 + 4x_3 &= 0 \\ -4x_1 - 4x_2 - 8x_3 &= 0 \\ -3x_2 - 3x_3 &= 0 \end{aligned}$$

Please turn over!

1.5.7 Let

$$A = \begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}$$

and describe all solutions of the matrix equation $A\mathbf{x} = \mathbf{0}$ in parametric form.

1.5.13 Suppose the solution set of a certain system of linear equations can be described as $x_1 = 5 + 4x_3$, $x_2 = -2 - 7x_3$, with x_3 free. Use vectors to describe this set as a line in $\mathbb{R}^3$

1.5.15 (1.5.17 in 3rd Ed.) Describe and compare the solution sets of $x_1 + 5x_2 - 3x_3 = 0$ and $x_1 + 5x_2 - 3x_3 = -2$