§ 7.3 Tables of Integrals

Using the table in the back of the book to find antiderivatives of certain functions.

Ex

$$\int \sin 4x \cos 3x \, dx$$

Product of two trig fns - use Part II of the table

II.12 $\quad a = 4, \quad b = 3$

$$\int \sin 4x \cos 3x \, dx = \frac{1}{3^2 - 4^2} (3 \sin 4x \sin 3x + 4 \cos 4x \cos 3x) + C$$

$$= -\frac{1}{7} (3 \sin 4x \sin 3x + 4 \cos 4x \cos 3x) + C$$

Ex.

$$\int (x^2+2x-1)e^{3x}\,dx$$

Of the form $\int p(x)\, e^{ax}\,dx$

where $p(x)$ is the poly. $p(x) = x^2+2x-1$.

Use III - 14 from the table.

$p(x) = x^2+2x-1$

$p'(x) = 2x+2$

$p''(x) = 2$

So

$$\int (x^2+2x-1)e^{3x}\,dx$$

$$= e^{3x}\left[\frac{1}{3}(x^2+2x-1) - \frac{1}{9}(2x+2) + \frac{1}{27}(2)\right] + C$$

Ex

$$\int \cos^3 t \sin^4 t \, dt$$

Trig fn where at least one of the powers is odd.

Using IV-23 rewrite this as

$$\int \cos^2 t \sin^4 t \cdot \cos t \, dt$$

and let $w(t) = \sin t$, so $dw = \cos t \, dt$.

Get

$$\int (1-w^2) \cdot w^4 \, dw \qquad \left[\cos^2 t = 1 - \sin^2 t = 1 - w^2 \right]$$

$$= \int (w^4 - w^6) \, dw$$

$$= \frac{w^5}{5} - \frac{w^7}{7} + C$$

$$= \frac{\sin^5 t}{5} - \frac{\sin^7 t}{7} + C$$

Ex. Find

$$\int \sin^4\theta \, d\theta$$

Trig fns with even powers - use IV-17 repeatedly.

$$\int \sin^4\theta \, d\theta = -\frac{1}{4}\sin^3\theta\cos\theta + \frac{3}{4}\int \sin^2\theta \, d\theta$$

$$\int \sin^2\theta \, d\theta = -\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\int 1 \, d\theta$$

Putting in for $\int \sin^2\theta \, d\theta$ in the first integral gives

$$\int \sin^4\theta \, d\theta = -\frac{1}{4}\sin^3\theta + \frac{3}{4}\left(-\frac{1}{2}\sin\theta\cos\theta + \frac{1}{2}\int 1 \, d\theta\right)$$

$$= -\frac{1}{4}\sin^3\theta - \frac{3}{8}\sin\theta\cos\theta + \frac{3}{8}\theta + C$$

n.b. no need to put in C until the end.

Often we need to prepare the integral before we can use the table

Ex. $\int \frac{x^2}{x^2+4}\,dx$

The integrand is a rational fn (i.e. a quotient of two polys.) where the degree of the numerator is $\geqslant$ that of the denominator.

The thing to do in this case is algebraic long division.

$$\begin{array}{r|l} & \;\;1 \\ \hline x^2+4 & x^2+0.x+0 \\ & \underline{x^2+0x+4} \\ & \qquad\quad -4 \end{array}$$

So $$\frac{x^2}{x^2+4} = 1 - \frac{4}{x^2+4}$$

and $$\int \frac{x^2}{x^2+4}\,dx$$

$$= \int \left(1 - \frac{4}{x^2+4}\right) dx$$

$$= \int dx - 4\int \frac{dx}{x^2+4}$$

$$= x - 4 \cdot \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C \qquad \text{by V-24}$$

$$= x - 2\arctan\left(\frac{x}{2}\right) + C$$

Another common trick is completing the square

e.g.

$$x^2 + 6x + 14$$

Want to write this in the form $(x+a)^2 + b$

ie

$$x^2 + 6x + 14 = (x+a)^2 + b$$

$$= x^2 + 2ax + a^2 + b$$

So comparing terms in x, we see

$$6 = 2a$$

So $a = 3$

and $x^2 + 6x + 14 = (x+3)^2 + b$

$$= x^2 + 6x + 9 + b$$

Then $14 = 9 + b$

So $b = 5.$

Thus.

$$x^2 + 6x + 14 = (x+3)^2 + 5.$$

Ex.

$$\int \frac{1}{x^2+6x+14}\, dx$$

Complete the square

$$= \int \frac{1}{(x+3)^2+5}\, dx$$

Let $w = x+3$, $dw = dx$ and rewrite

$$\int \frac{1}{w^2+5}\, dw$$

$$= \frac{1}{\sqrt{5}} \arctan\left(\frac{w}{\sqrt{5}}\right) + C$$

Convert back to x

$$= \frac{1}{\sqrt{5}} \arctan\left(\frac{x+3}{\sqrt{5}}\right) + C$$

Ex

$$\int e^t \sin(5t+7)\,dt$$

Looks similar to II-8, but not quite the same.

Let $w = 5t+7$, $dw = 5\,dt$.

So $t = \frac{w-7}{5}$, $dt = \frac{dw}{5}$

Rewrite

$$\int e^{\frac{w-7}{5}} \sin w \,\frac{dw}{5}$$

$$= \frac{1}{5} e^{-7/5} \int e^{\frac{w}{5}} \sin w \, dw$$

Now apply II-8 with $a = \frac{1}{5}$, $b = 1$ to get

$$= \frac{1}{5} e^{-7/5} \left[\frac{1}{(\frac{1}{5})^2 + 1^2} e^{w/5} \left(\frac{\sin w}{5} - \cos w \right) \right] + C$$

$$= \frac{1}{5} e^{-7/5} \cdot \frac{25}{26} e^{w/5} \cdot \left(\frac{1}{5} \sin w - \cos w\right) + C$$

$$= \frac{5}{26} e^{\frac{w-7}{5}} \cdot \left(\frac{1}{5} \sin w - \cos w\right) + C$$

Convert back to t

$$= \frac{5}{26} e^{t}\left(\frac{\sin(5t+7)}{5} - \cos(5t+7)\right) + C$$

$$= \frac{e^{t}}{26}\left(\sin(5t+7) - 5\cos(5t+7)\right) + C$$