# Math 142

# Second Semester Calculus

Much of the earlier part of the course will be devoted to techniques for doing integration, so we begin with some

## Review

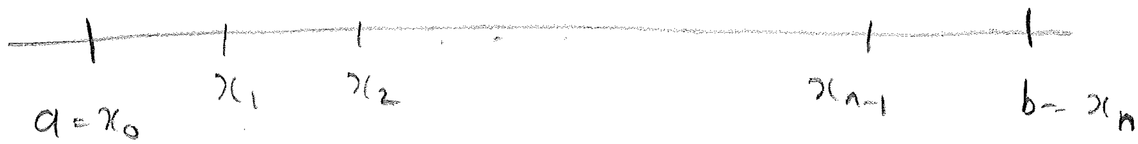
### Definite Integrals

Let $f(x)$ be a function ($f^n$) defined on the closed interval $[a,b] = \{x \in \mathbb{R} ; a \le x \le b\}$.

Divide $[a,b]$ into $n$ equal subintervals of width

$$\Delta x = \frac{b-a}{n}$$

which are disjoint except for their endpts.

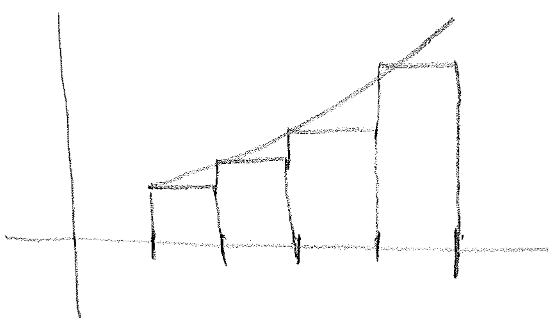



For $1 \leq i \leq n$, the $i$th interval is given by

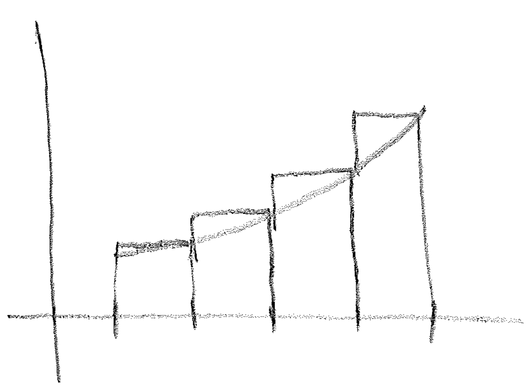
$$I_i = [x_{i-1}, x_i] = [a + (i-1)\Delta x, a + i\Delta x].$$

Form the

Left-hand Riemann Sum $\sum_{i=0}^{n-1} f(x_i)\Delta x$



and the right-hand Riemann Sum $\sum_{i=1}^{n} f(x_i)\Delta x$



Now take the limit as the number of intervals, $n$, tends to infinity.

If the limits for both left- and right-hand sums exist and are equal, we say that $f$ is <u>integrable</u> on $[a,b]$ and we call the value of this limit the <u>definite integral</u> of $f$ over $[a,b]$, written

$$\int_a^b f(x)\,dx.$$

<u>FACT</u> If $f$ is continuous (cts.) on $[a,b]$, then $f$ is integrable and

$$\int_a^b f(x)\,dx$$

exists.

# Antiderivatives

$F(x)$ is an antiderivative of $f(x)$ if
$F'(x) = f(x)$.

Antidifferentiation is basically the opposite of differentiation

$$F \xrightarrow{\text{diff.}} f$$

$$f \xrightarrow{\text{antid.}} F$$

FACTS.

1. Adding a constant to an antiderivative (antid.) gives another antid. of the same function.

2. Any two antids. of the same fn differ by a constant.

Since all the possible antids. of a fn $f$ are so similar, we can add an arbitrary unspecified constant to one of them to describe all of them simultaneously.

This gives us the notion of the most general antiderivative of $F$, also called the <u>indefinite integral</u> of $f$, written

$$F(x) + C \quad \text{or} \quad \int f(x)\,dx \quad \text{where}$$

$C$ is the arbitrary constant.

Note that $\int_a^b f(x)\,dx$ and $\int f(x)\,dx$ are very different as objects, even though they are written so similarly.

$\int_a^b f(x)\,dx$ is a <u>number</u>

$\int f(x)\,dx$ is a <u>family of functions</u> (differing by constants).

The two objects are related by the <u>Fundamental Theorem of Calculus</u> (FTOC).

# Fundamental Theorem of Calculus - First Part

(Thm 5.1, P. 256)

If $f$ is cts on $[a,b]$ and $F$ is an antid. of $f$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Allows us to compute definite integrals without resorting to Riemann sums.

e.g. $x^3$ is an antid of $3x^2$ as
$\frac{d}{dx}(x^3) = 3x^2$.

Thus by FTOC (I),

$$\int_1^2 3x^2\,dx = \left[x^3\right]_1^2 = 2^3 - 1^3 = 7.$$

Finally we have

# Fundamental Theorem of Calculus - Second Part

(Thm 6.2 P. 299)

If $f$ is cts. on an interval and $a$ is any pt. in that interval, then the function $F$ given by

$$F(x) = \int_a^x f(t)\,dt$$

is an antid. of $f$.

In other words,

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x).$$

'Get rid of the $\int$ sign and convert $t$ to $x$'.

The two parts of FTOC show how differentiation and integration are related.