# Math 142 Worksheet # 1 Solutions

1. i) $\int x e^{-x^2} dx$

$w = -x^2, \quad dw = -2x\,dx$

$x\,dx = -\frac{dw}{2}$

$$\int x e^{-x^2} dx = \int e^{w} \cdot -\frac{dw}{2}$$

$$= -\frac{1}{2} \int e^{w} dw$$

$$= -\frac{1}{2} e^{w} + C$$

$$= -\frac{1}{2} e^{-x^2} + C$$

ii) $$\int_0^1 \frac{x}{1+x^2}\,dx$$

$w = 1+x^2$, $dw = 2x\,dx$

$x\,dx = \frac{dw}{2}$

<u>Limits</u> when $x=0$, $w=1$

$x=1$, $w=2$

Get $$\int_1^2 \frac{1}{2}\frac{dw}{w} = \frac{1}{2}\int_1^2 \frac{dw}{w} = \frac{1}{2}\left[\ln|w|\right]_1^2$$

$$= \frac{1}{2}(\ln 2 - \ln 1)$$

$$= \frac{1}{2}\ln 2.$$

iii) $$\int_0^{\frac{\pi}{4}} \frac{\sin(\tan x)}{\cos^2 x}\,dx = \int_0^{\frac{\pi}{4}} \sin(\tan x)\cdot \sec^2 x\,dx$$

Let $w = \tan x$, $dw = \sec^2 x\,dx$.

When $x=0$, $w=0$

$x=\frac{\pi}{4}$, $w=1$

$$= \int_0^1 \sin w\,dw = \left[-\cos w\right]_0^1 = -\cos 1 - (-\cos 0)$$

$$= 1 - \cos 1$$

iv) $$\int x(x^2-4)^{7/2}\,dx$$

$w = x^2-4 \qquad dw = 2x\,dx$

$\qquad\qquad\qquad \frac{dw}{2} = x\,dx.$

$$\int x(x^2-4)^{7/2}\,dx = \int w^{7/2}\cdot \frac{dw}{2}$$

$$= \frac{1}{2}\int w^{7/2}\cdot dw$$

$$= \frac{1}{2}\,\frac{w^{9/2}}{\frac{9}{2}} + C$$

$$= \frac{w^{9/2}}{9} + C$$

$$= \frac{(x^2-4)^{9/2}}{9} + C$$

2. i) $$\int_0^{\frac{\pi}{2}} x\cos(2x)\,dx$$

Let $u = x$ , $dv = \cos(2x)\,dx$

$du = dx$ , $v = \frac{1}{2}\sin(2x)$.

$$= \left[x\cdot\frac{1}{2}\sin(2x)\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}}\frac{1}{2}\sin(2x)\cdot dx.$$

$$= \left[\frac{x\sin(2x)}{2}\right]_0^{\frac{\pi}{2}} - \frac{1}{2}\int_0^{\frac{\pi}{2}}\sin(2x)\,dx.$$

$$= \frac{1}{2}\left(\frac{\pi}{2}\sin\pi - 0\right) - \frac{1}{2}\left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$

$$= 0 - \frac{1}{2}\left(-\frac{1}{2}\cos\pi - \left(-\frac{1}{2}\cos 0\right)\right)$$

$$= -\frac{1}{2}\left(-\frac{1}{2}(-1) - \left(-\frac{1}{2}\right)(1)\right)$$

$$= -\frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\right) = -\frac{1}{2}.$$

ii) $$\int_0^1 x^2 e^{3x}\,dx$$

$u = x^2$ , $dv = e^{3x}\,dx$

$du = 2x\,dx$ , $v = \frac{1}{3}e^{3x}$

$$= \left[x^2 \cdot \frac{1}{3}e^{3x}\right]_0^1 - \int_0^1 \frac{1}{3}e^{3x} \cdot 2x\,dx$$

$$= \left[\frac{x^2 e^{3x}}{3}\right]_0^1 - \frac{2}{3}\int_0^1 x e^{3x}\,dx.$$

$u = x$ , $dv = e^{3x}$

$du = dx$ , $v = \frac{1}{3}e^{3x}$

$$= \frac{e^3}{3} - 0 - \frac{2}{3}\left\{\left[\frac{xe^{3x}}{3}\right]_0^1 - \int_0^1 \frac{1}{3}e^{3x}\,dx\right\}$$

$$= \frac{e^3}{3} - \frac{2}{3}\left\{\frac{e^3}{3} - \left[\frac{1}{9}e^{3x}\right]_0^1\right\}$$

$$= \frac{e^3}{3} - \frac{2}{3}\left\{\frac{e^3}{3} - \left(\frac{e^3}{9} - \frac{1}{9}\right)\right\}$$

$$= \frac{e^3}{3} - \frac{2e^3}{9} + \frac{2e^3}{27} - \frac{2}{27} = \frac{5e^3}{27} - \frac{2}{27}.$$

iii) $\int x^2 \ln x \, dx$

$u = \ln x$ , $dv = x^2 dx$

$du = \frac{dx}{x}$ , $v = \frac{x^3}{3}$.

$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{dx}{x}$

$= \frac{x^3 \ln x}{3} - \frac{1}{3} \int x^2 dx$

$= \frac{x^3 \ln x}{3} - \frac{1}{3} \cdot \frac{x^3}{3} + C$

$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C.$

iv) $\int e^{-x} \cos(3x)dx$.

$u = e^{-x}$, $dv = \cos(3x)dx$

$du = -e^{-x}dx$, $v = \frac{1}{3}\sin(3x)$

$= e^{-x} \cdot \frac{1}{3}\sin(3x) - \int \frac{1}{3}\sin(3x) \cdot -e^{-x} dx$

$= \frac{1}{3}e^{-x}\sin(3x) + \frac{1}{3}\int e^{-x}\sin(3x)dx$

$u = e^{-x}$, $dv = \sin(3x)dx$

$du = -e^{-x}dx$, $v = -\frac{1}{3}\cos(3x)$

$= \frac{1}{3}e^{-x}\sin(3x) + \frac{1}{3}\left\{ e^{-x} \cdot -\frac{1}{3}\cos(3x) - \int -\frac{1}{3}\cos(3x) \cdot -e^{-x}dx \right\}$

$= \frac{1}{3}e^{-x}\sin(3x) - \frac{1}{9}e^{-x}\cos(3x) - \frac{1}{9}\underline{\int e^{-x}\cos(3x)dx}$

same as
original integral.

Add $\frac{1}{9}\int e^{-x}\cos(3x)\,dx$ to both sides.

$$\frac{10}{9}\int e^{-x}\cos(3x)\,dx = \frac{1}{3}e^{-x}\sin(3x) - \frac{1}{9}e^{-x}\cos(3x) + C'$$

So

$$\int e^{-x}\cos(3x)\,dx = \frac{3}{10}e^{-x}\sin(3x) - \frac{1}{10}e^{-x}\cos(3x) + C,$$

$$C = \frac{9C'}{10}$$

v). $\int \sin(2x)\cos(3x)\,dx.$

Let $u = \sin(2x)$, $dv = \cos(3x)\,dx$

$du = 2\cos(2x)\,dx$, $v = \frac{1}{3}\sin(3x)$.

$$= \sin(2x)\cdot\frac{1}{3}\sin(3x) - \int \frac{1}{3}\sin(3x)\cdot 2\cos(2x)\,dx$$

$$= \frac{1}{3}\sin(2x)\sin(3x) - \frac{2}{3}\int \cos(2x)\sin(3x)\,dx$$

Now let $u = \cos(2x)$, $dv = \sin(3x)\,dx$

$du = -2\sin(2x)\,dx$, $v = -\frac{1}{3}\cos(3x)$.

$$= \frac{1}{3}\sin(2x)\sin(3x) - \frac{2}{3}\left\{\cos(2x)\cdot\frac{1}{3}\cos(3x) - \int \frac{1}{3}\cos(3x)\cdot -2\sin(2x)\,dx\right\}$$

$$= \frac{1}{3}\sin(2x)\sin(3x) - \frac{2}{9}\cos(2x)\cos(3x) - \frac{4}{9}\underbrace{\int \sin(2x)\cos(3x)\,dx}_{\text{original integral.}}$$

Add $\frac{4}{9}\int \sin(2x)\cos(3x)$ to both sides.

$$\frac{13}{9}\int \sin(2x)\cos(3x)\,dx = \frac{1}{3}\sin(2x)\sin(3x) - \frac{2}{9}\cos(2x)\cos(3x) + C'$$

$$\int \sin(2x)\cos(3x)\,dx = \frac{3}{13}\sin(2x)\sin(3x) - \frac{2}{13}\cos(2x)\cos(3x) + C,$$

$$C = \frac{9C'}{13}.$$