Section 5.1: How Do We Measure Distance Traveled?

A thought Experiment: How Far Did the Car Go?

<table>
<thead>
<tr>
<th>time (sec)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity (ft/sec)</td>
<td>20</td>
<td>30</td>
<td>38</td>
<td>44</td>
<td>48</td>
<td>50</td>
</tr>
</tbody>
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![Diagram of velocity over time with area estimates and differences between estimates.](image)
Visualizing Distance on the Velocity Graph: Area Under Curve

Velocity measured every 1/2 second:

Velocity measured every 1/4 second:

Distance traveled is area under curve:
EX: With time $t$ in seconds, the velocity of a bicycle, in feet per second, is given by $v(t) = 5t$. How far does the bicycle travel in 3 seconds?

![Graph showing velocity over time]

EX: A particle moves along the $y$-axis with velocity 30 cm/sec for 5 seconds and velocity $-10$ cm/sec for the next 5 seconds. Positive velocity indicates upward motion; negative velocity represents downward motion. What is the net change in position of the particle?

![Graph showing velocity over time for particle motion]
Left and Right Sums

Left-hand sums:

Right-hand sums:

Left and right sums if $f$ is decreasing:
Problem 2 The velocity $v(t)$ in the table below is decreasing, $2 \leq t \leq 12$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v(t)$</td>
<td>44</td>
<td>42</td>
<td>41</td>
<td>40</td>
<td>37</td>
<td>35</td>
</tr>
</tbody>
</table>

Using $n = 5$ subdivisions to approximate the total distance traveled, find

(a) An upper estimate

(b) A lower estimate
Problem 6 Use the expressions for left and right sums and the table.

(a) If \( n = 4 \), what is \( \Delta t \) What are \( t_0, t_1, t_2, t_3, t_4 \)? What are \( f(t_0), f(t_1), f(t_2), f(t_3), f(t_4) \)?

<table>
<thead>
<tr>
<th>( t )</th>
<th>15</th>
<th>17</th>
<th>19</th>
<th>21</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(t) )</td>
<td>10</td>
<td>13</td>
<td>18</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

(b) Find the left and right sums using \( n = 4 \).

(c) If \( n = 2 \), what is \( \Delta t \) What are \( t_0, t_1, t_2 \)? What are \( f(t_0), f(t_1), f(t_2) \)?

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(d) Find the left and right sums using \( n = 2 \).
Problem 8 At time, $t$, in seconds, your velocity, $v$, in meters/second, is given by $v(t) = 1 + t^2$ for $0 \leq t \leq 6$. Use $\Delta t = 2$ to estimate the distance traveled during this time. Find the upper and lower estimates, and then average the two.

Exercise 14 shows the velocity, in cm/sec, of a particle moving along the $x$-axis. Compute the particle’s change in position, left (negative) or right (positive), between times $t = 0$ and $t = 5$ seconds.

Problem 14
In Problem 18, find the difference between the upper and lower estimates of the distance traveled at velocity $f(t)$ on the interval $a \leq t \leq b$ for $n$ subdivisions.

Problem 18 $f(t) = 25 - t^2$, $a = 1$, $b = 4$, $n = 500$

Problem 22 A baseball thrown directly upward at 96 ft/sec has velocity $v(t) = 96 - 32t$ ft/sec at time $t$ seconds.

(a) Graph the velocity from $t = 0$ to $t = 6$.

(b) When does the baseball reach the peak of its flight? How high does it go?

(c) How high is the baseball at time $t = 5$?