Alternating 4-cycles in graphs

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Department of Mathematics
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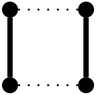
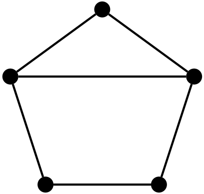
Joint work with Douglas B. West

Alternating 4-cycles...

Alternating 4-cycle (A_4)





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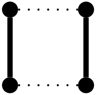

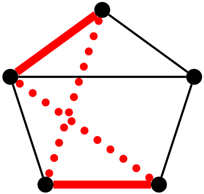

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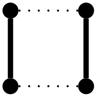

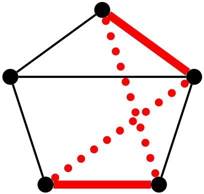

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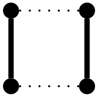

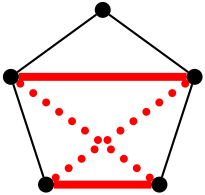

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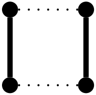

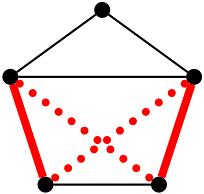

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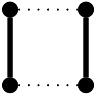
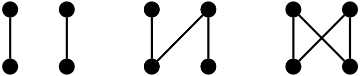
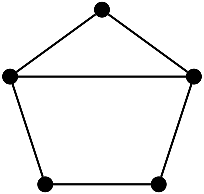


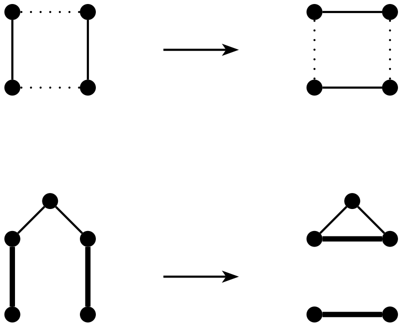


$2K_2$ P_4 C_4



...and degree sequences

2-switches



Theorem (Fulkerson–Hoffman–McAndrew, 1965)

$\deg(G) = \deg(H)$ *iff 2-switches transform G into H.*

...and graph classes

- Threshold graphs (Chvátal–Hammer, 1973)
  No A_4's present.

- Matrogenic graphs (Földes–Hammer, 1976)
  Vertex sets of A_4's are circuits of a matroid on V.

- Matroidal graphs (Peled, 1977)
  Edge sets of A_4's are circuits of a matroid on E.

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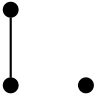

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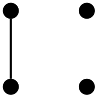

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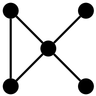

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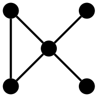

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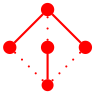

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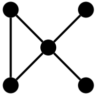

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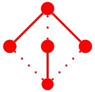

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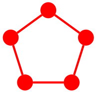


Questions

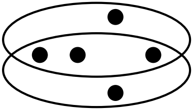
What structural properties of a graph can we tie to the **existence** and **location** of alternating 4-cycles?

How do these affect the degree sequence?

The A_4-structure of a graph

Hypergraph H

$$V(H) = V(G), \qquad E(H) = \{A \subseteq V(G) : G[A] \cong 2K_2 \text{ or } C_4 \text{ or } P_4\}$$



Characterizations in terms of the A_4-structure

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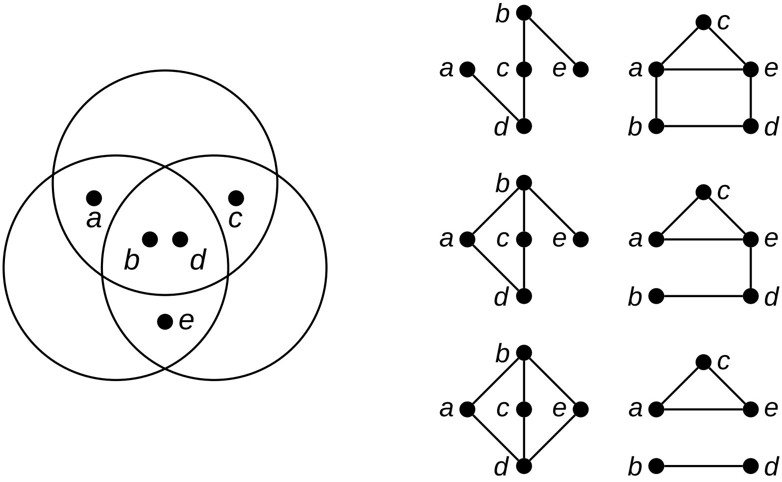
Characterizations in terms of the A_4-structure

- Threshold graphs (Chvátal–Hammer, 1973)
  No A_4's present.
  The A_4-structure has no edges.

- Matrogenic graphs (Földes–Hammer, 1976)
  Vertex sets of A_4's are circuits of a matroid on V.
  No 5 vertices induce exactly 2 or 3 edges in the A_4-structure.

- Matroidal graphs (Peled, 1977)
  Edge sets of A_4's are circuits of a matroid on E.
  No 5 vertices induce more than 1 edge in the A_4-structure.

Graphs with a common A_4-Structure



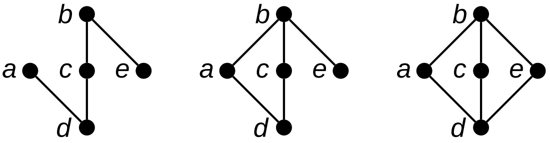
What properties does the A_4-structure determine?

Matchings

A *nontrivial matching* is a set of at least two pairwise non-intersecting edges.

Theorem

Let G and H be triangle-free graphs with the same vertex set V. If G and H have the same A_4-structure and $W \subseteq V$, then W is the vertex set of a nontrivial matching in one of these graphs if and only if it is in the other.



Corollary

Two triangle-free graphs with the same A_4-structure have maximum matchings of the same size.

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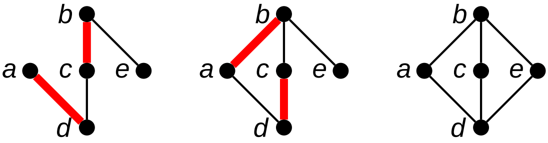

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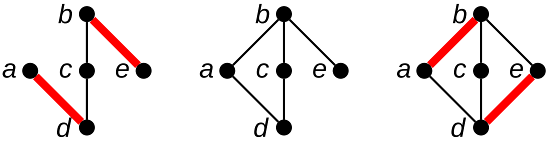

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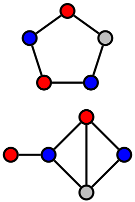
Perfect graphs

The *chromatic number* of a graph is the minimum number of colors needed to properly color the graph.



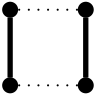
The *clique number* is the size of largest set of pairwise adjacent vertices.

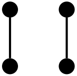
A graph is *perfect* if in every induced subgraph the chromatic number equals the clique number.

Theorem

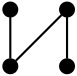
If G and H have the same A_4-structure, then G is perfect iff H is perfect.

P_4 and modules





$2K_2$



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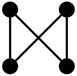

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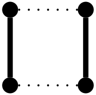
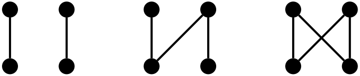
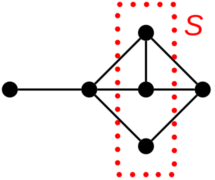




$2K_2$ P_4 C_4

A *module* is a vertex subset S such that each vertex outside S is joined to S by either all possible edges or no edges.



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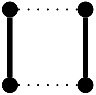

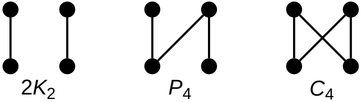

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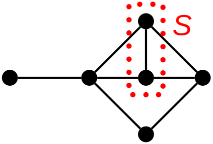

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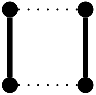

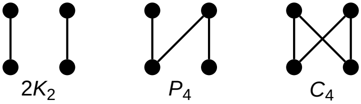

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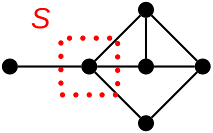

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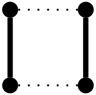

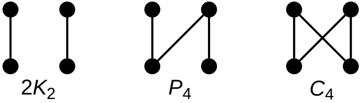

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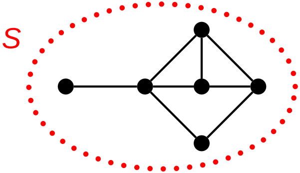

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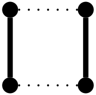

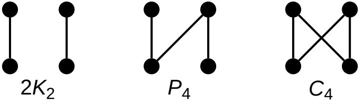

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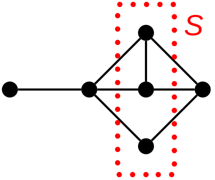

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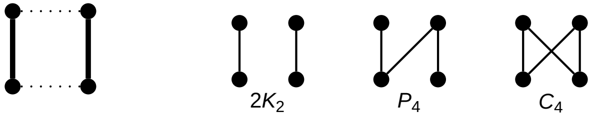

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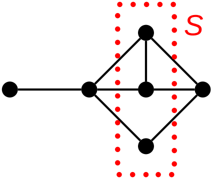

Theorem

- *An induced P_4 intersects a module in exactly 0, 1, or 4 vertices.*
- (Seinsche, 1974) *In a graph G every induced subgraph on at least 3 vertices contains a nontrivial module iff G is P_4-free.*

Modules

A *module* S is a vertex subset such that no alternating path of length 2 begins and ends in S and has its midpoint outside S.

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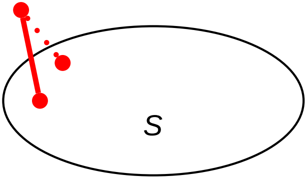

Strict modules

Define a **strict module** to be a vertex subset S such that no (**possibly closed**) alternating path of length 2 **or 3** begins and ends in S and has its midpoints outside S.

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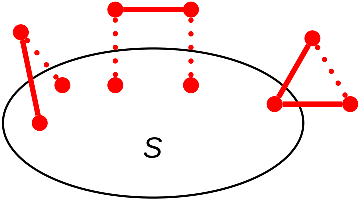

This is equivalent to not having alternating paths of *any* length begin and end in S.

A_4 and strict modules

Lemma

An A_4 intersects a strict module in exactly 0 or 4 vertices.

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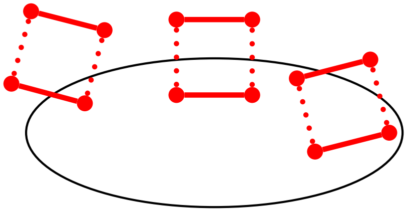

Theorem

In a graph G every induced subgraph on at least 2 vertices has a nontrivial strict module if and only if G is A_4-free, i.e., threshold.

Strict modules and graph structure

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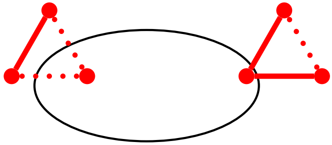
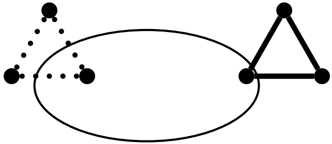


Strict modules and graph structure

Lemma

The vertices which dominate a strict module form a clique, and those which are nonadjacent to the strict module form an independent set.



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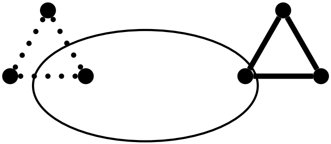
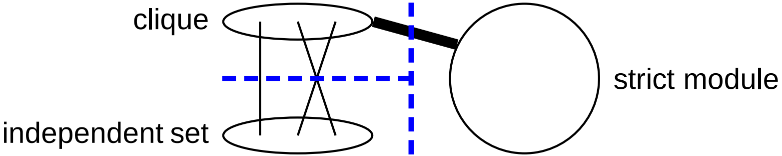
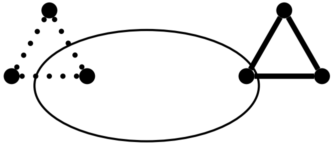


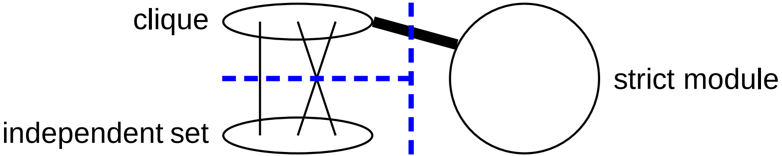


Strict modules and graph structure

Lemma

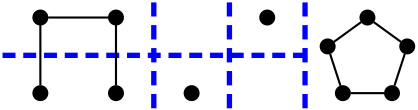
The vertices which dominate a strict module form a clique, and those which are nonadjacent to the strict module form an independent set.





Iterate to get a “strict modular decomposition”?

Decomposition



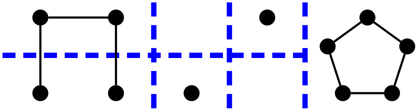
Canonical decomposition

Theorem (Tyshkevich–Chernyak, 1978; Tyshkevich, 2000)

Every graph F can be represented as a composition

$$F = (G_k, A_k, B_k) \circ \cdots \circ (G_1, A_1, B_1) \circ F_0$$

of indecomposable components. Here the (G_i, A_i, B_i) are indecomposable splitted graphs and F_0 is an indecomposable graph. This decomposition is unique up to isomorphism of components.



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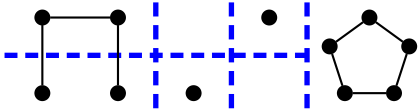
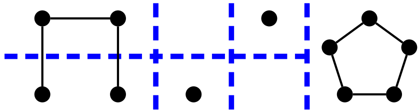


Where did this come from?

Previous motivation for canonical decomposition



Just seemed to show up...

- Matrogenic graphs (Földes–Hammer, 1976; Tyshkevich, 1984)
- Unigraphs (Tyshkevich–Chernyak, 1978–1979)
- Box-threshold graphs (Tyshkevich–Chernyak, 1985)
- Pseudo-split graphs (Blázsik et al., 1993)

In each case, indecomposable components restricted to certain classes.

Structural properties lead to degree sequence characterizations.

A_4 and strict modules

Lemma

An A_4 intersects a strict module in exactly 0 or 4 vertices.

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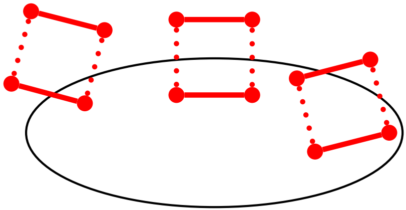


Theorem

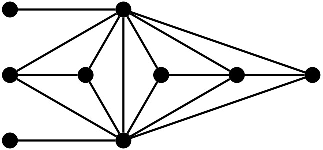
In a graph G every induced subgraph on at least 2 vertices has a nontrivial strict module if and only if G is A_4-free, i.e., threshold.

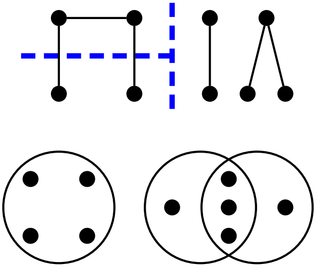
A_4 and canonical decomposition

Theorem

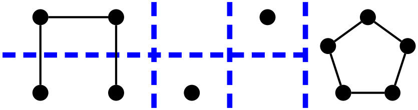
A graph is indecomposable in the canonical decomposition if and only if its A_4-structure is connected.

Hence the components of the A_4-structure and of the canonical decomposition partition the vertex set in the same way.





Motivation for canonical decomposition



- Graph classes (matrogenic, unigraphs, etc.)
- Strict modular decomposition
- Components of the A_4-structure

A_4 and canonical decomposition: a proof

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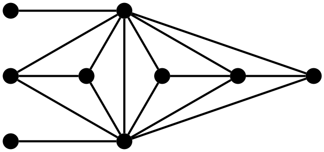

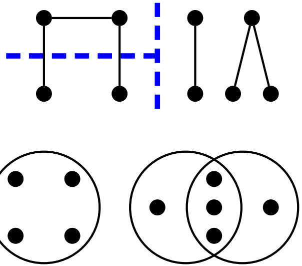

Beginnings

Lemma

The graphs $2K_2$, C_4, and P_4 are all indecomposable. Therefore, connected A_4-structure $\implies$ indecomposable.

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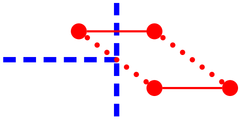


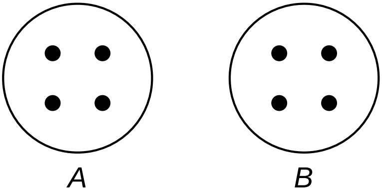
Lemma

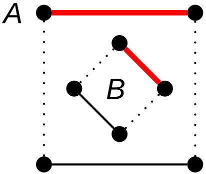
In an indecomposable graph G with more than 1 vertex, every vertex belongs to an alternating 4-cycle.

Disjoint A_4s

Lemma

If A and B are disjoint alternating 4-cycles in G such that no third alternating cycle in G intersects each, then either A induces P_4, with its interior vertices dominating B and the endpoints isolated from B (denote this by $A \to B$), or vice versa.





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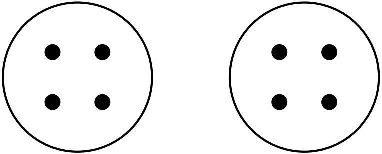
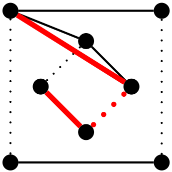


A B



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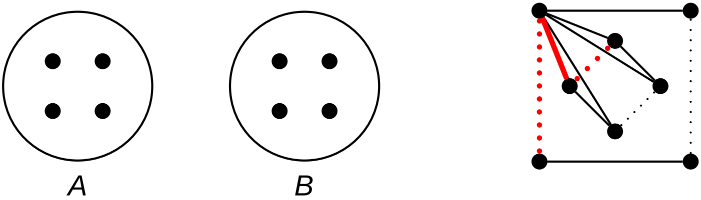

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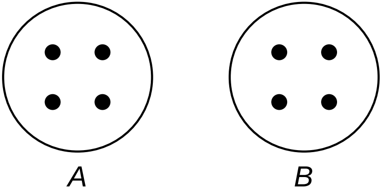

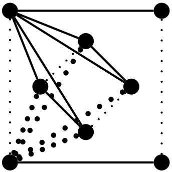

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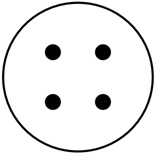

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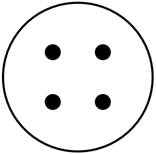

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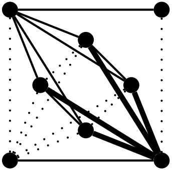

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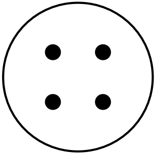

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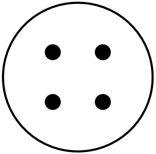

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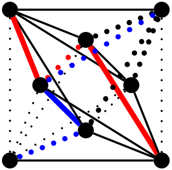

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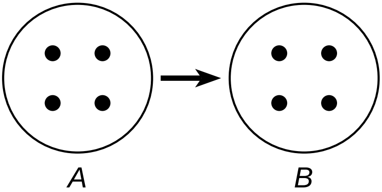
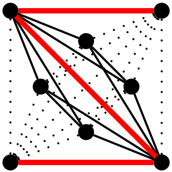
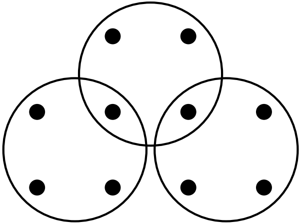


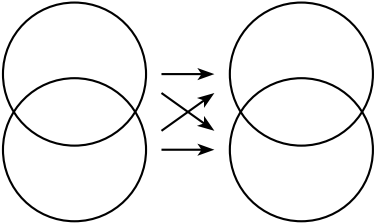


More on disjoint A_4s

Corollary

Any two vertices which both belong to induced $2K_2$'s or C_4's have distance at most 3 in the A_4-structure.





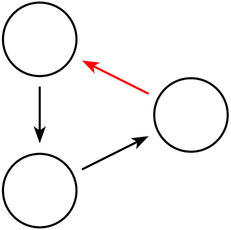
Lemma

The $\rightarrow$ relation is consistent among pairs of A_4s from different components of the A_4-structure.

Putting it all together

Lemma

The $\rightarrow$ tournament on the A_4-components of a graph is acyclic.



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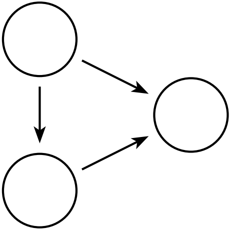

Having a source implies the graph is decomposable.

$$\therefore \text{not } A_4\text{-connected} \implies \text{decomposable.}$$

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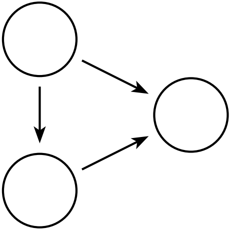


Having a source implies the graph is decomposable.

$$\therefore A_4\text{-connected} \iff \text{indecomposable.}$$

□

Questions

What structural properties of a graph can we tie to the **existence** and **location** of alternating 4-cycles?

How do these affect the degree sequence?

Degree sequence connections

Theorem (Erdős–Gallai, 1960)

Let $d = (d_1, \ldots, d_n)$ be a list of nonnegative integers with even sum, arranged in descending order. d is the degree sequence of a simple graph if and only if for all k,

$$\sum_{i \le k} d_i \le k(k-1) + \sum_{i > k} \min\{k, d_i\}.$$

Degree sequence connections

Theorem (B, 2013)

Let d be the degree sequence of G. The graph G is canonically indecomposable if and only if $d_n > 0$ and no Erdős–Gallai inequality holds with equality.

Moreover, by examining the values k for which the kth inequality is an equality, we can determine the sizes of the "cells" in the canonical decomposition.

Corollary

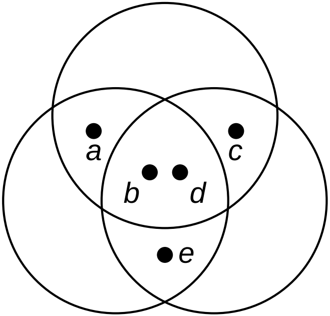
Knowing which Erdős–Gallai inequalities hold with equality (and the multiplicity of 0 as a term in d) is equivalent to knowing the vertex sets of the A_4-structure components.

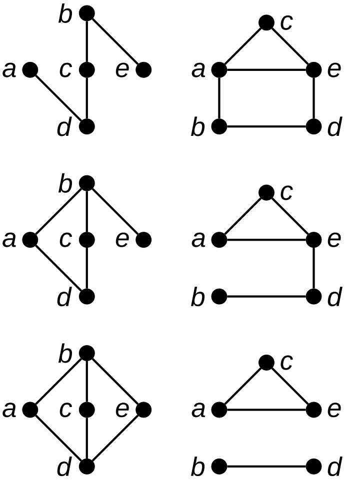
Future applications of the A_4-structure

Characterizations of graph/degree sequence properties

- Graph classes (threshold, matrogenic, etc.)
- Matchings
- Perfection
- Strict modules/canonical decomposition
- Erdős–Gallai inequalities
- ?

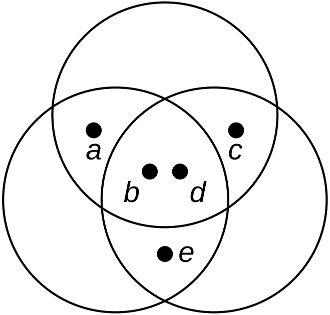
Graphs with a common A_4-Structure

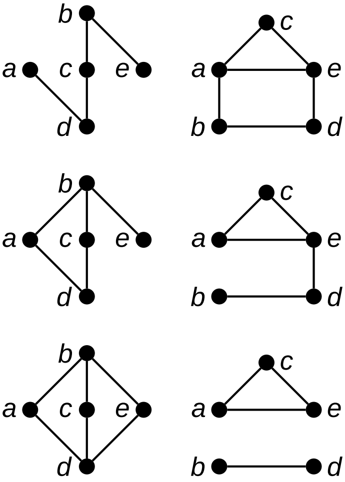




What other properties does the A_4-structure determine?

Graphs with a common A_4-Structure





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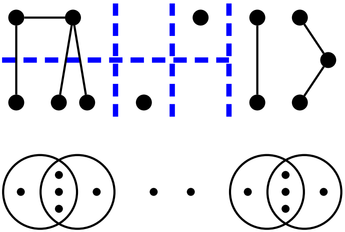
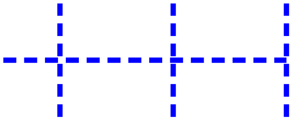
Which graphs have the same A_4-structure?

Obtaining other realizations: decomposable graphs





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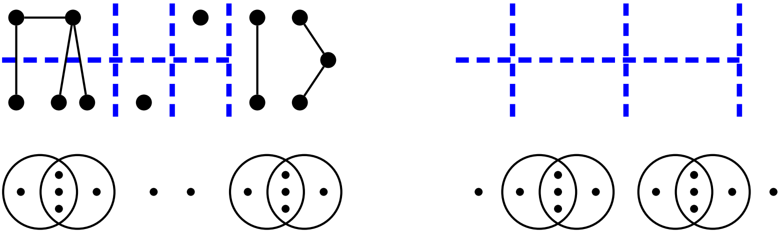

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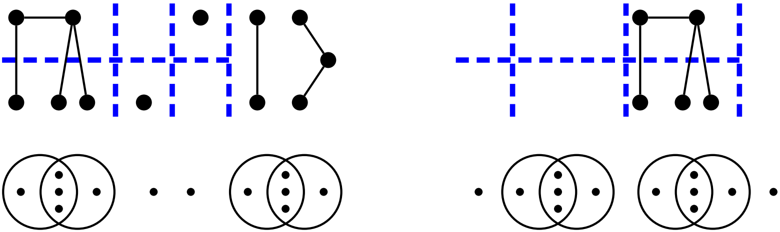

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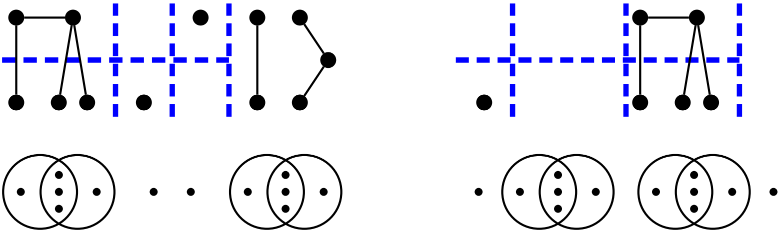

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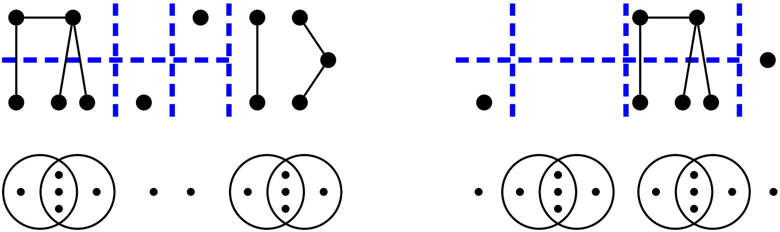

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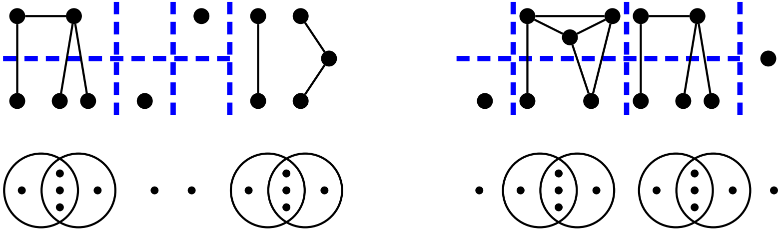
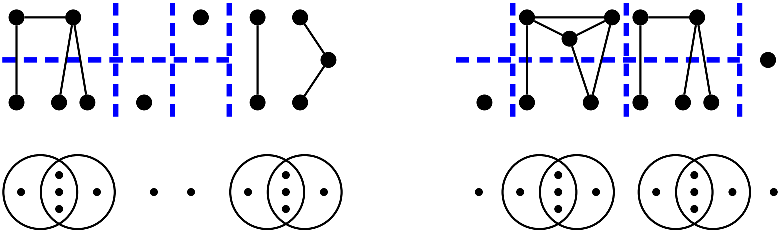


Obtaining other realizations: decomposable graphs



The rightmost A_4-component may only be transposed if it has a split realization.

Which graphs have the same A_4-structure as a split graph?

A_4-split graphs

Theorem

A graph is A_4-split iff each canonical component is. For an indecomposable graph G with A_4-structure H, the following are equivalent:

(i) *G is A_4-split.*

(ii) *H is balanced and satisfies the bipartite restriction property.*

(iii) *G is $\{C_5, P_5, house, K_2 + K_3, K_{2,3}, P, \overline{P}, K_2 + P_4, P_4 \vee 2K_1, K_2 + C_4, 2K_2 \vee 2K_1\}$-free.*

(iv) *G is split, or G or $\overline{G}$ is a disjoint union of stars.*

(v) *G is A_4-separable.*

Future applications of the A_4-structure

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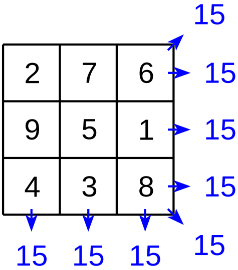
Antimagic labelings of graphs



Magic square: equal sums along each row, column, and main diagonal.

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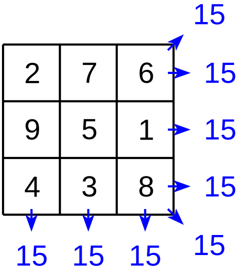

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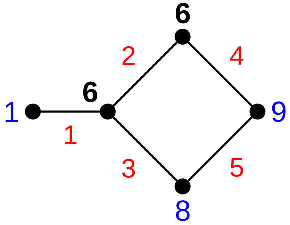
Conjecture (Hartsfield–Ringel, 1990): Every connected graph other than K_2 has an antimagic labeling.



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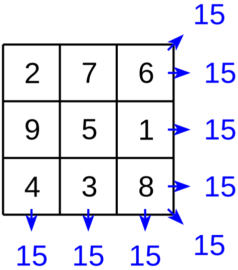

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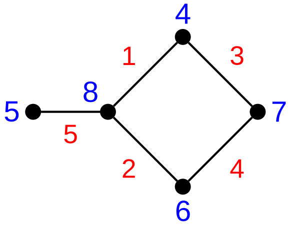
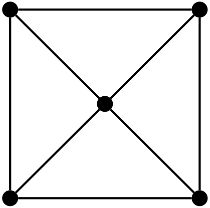


Canonical decomposition?

Theorem (Alon et al., 2004)

If G ($\not\cong K_2$) has a vertex which is adjacent to all other vertices, then G has an antimagic labeling.

Pf:



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If G ($\not\cong K_2$) has a vertex which is adjacent to all other vertices, then G has an antimagic labeling.

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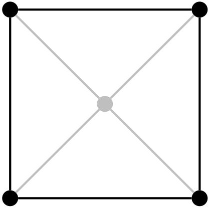

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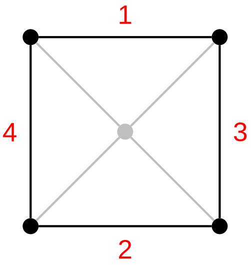

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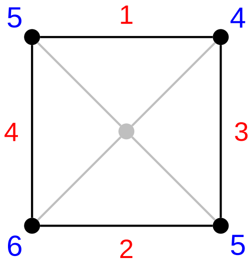

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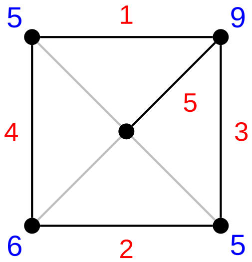

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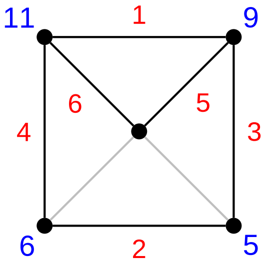

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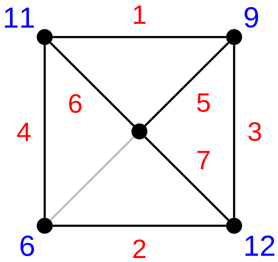

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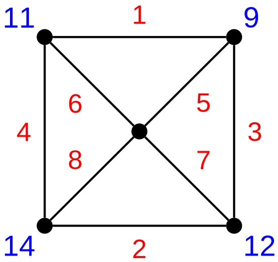

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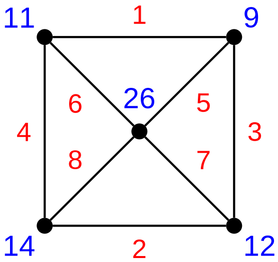

□

Success

Theorem (B, 2010)

If connected G ($\not\cong K_2$) is split or canonically decomposable, then G has an antimagic labeling.

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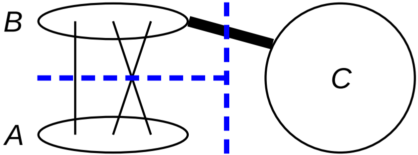

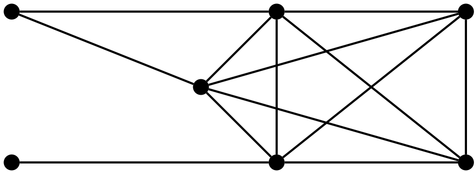

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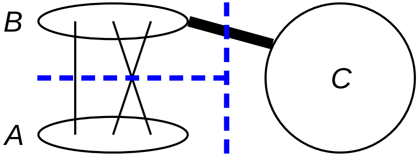

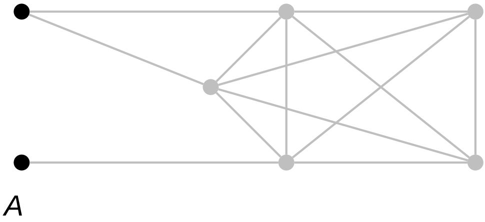

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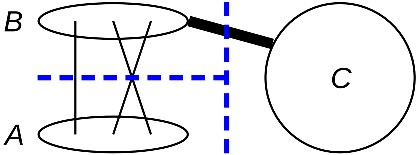

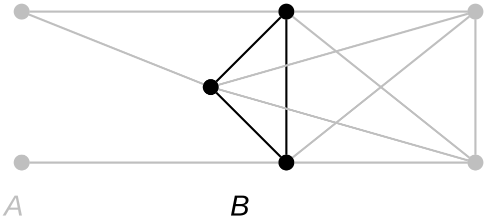

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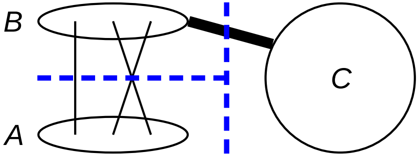

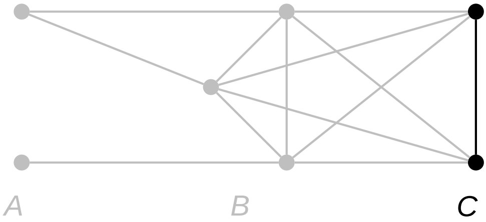

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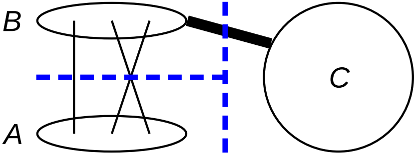

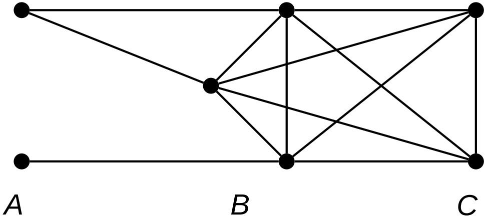

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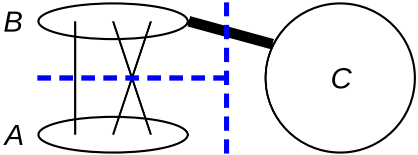

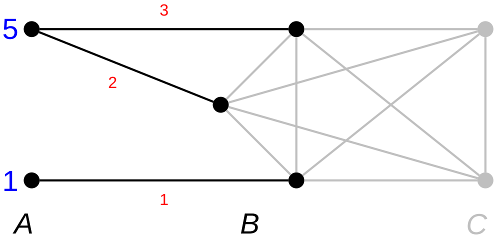

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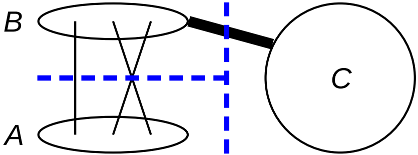

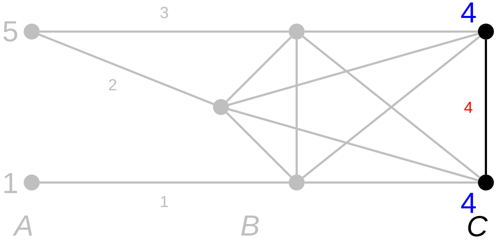

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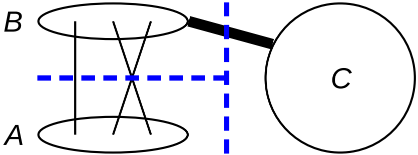

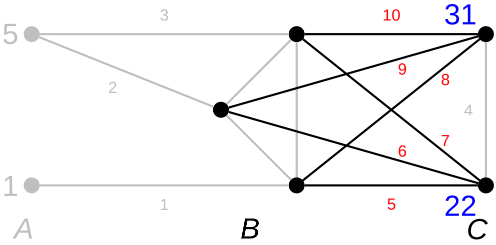

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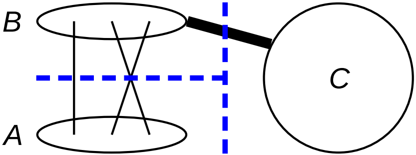

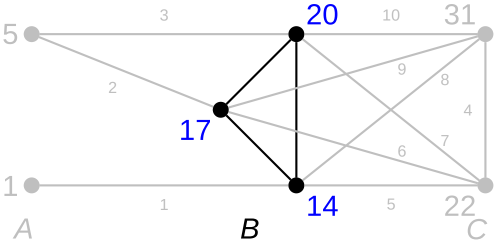

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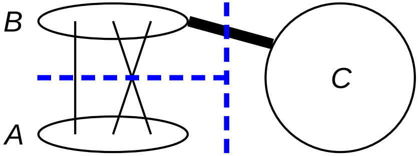

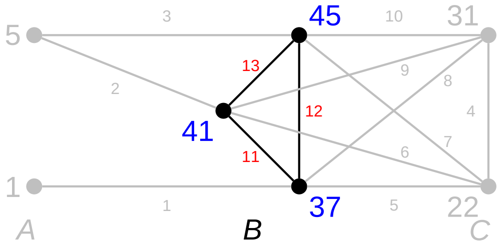

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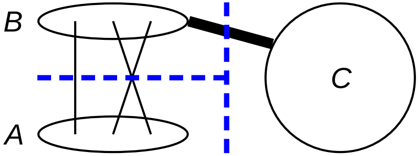

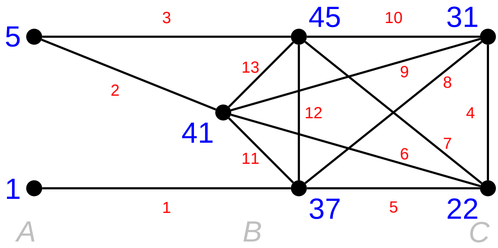

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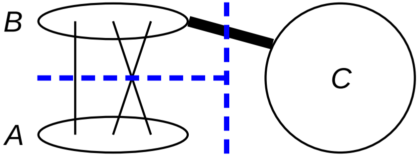

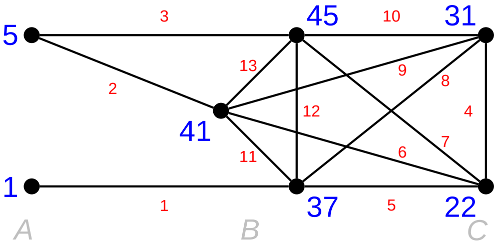

Possible A_4-structure help?

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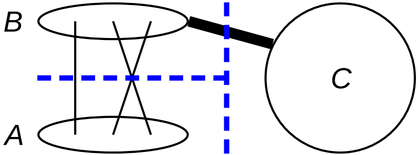

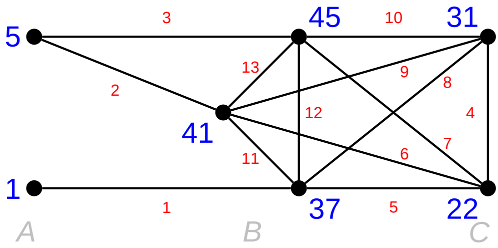

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True conjecture: how to label!

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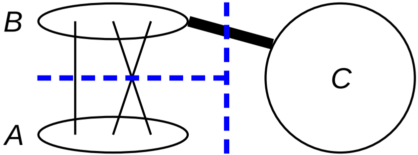

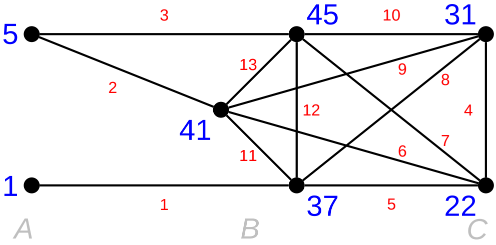

Possible A_4-structure help?
True conjecture: how to label!

False conjecture: counterexample!

Future applications of the A_4-structure

Characterizations of graph/degree sequence properties

- Graph classes (threshold, matrogenic, etc.)
- Matchings
- Perfection
- Strict modules/canonical decomposition
- Erdős–Gallai inequalities
- ?